Terms of Trade Gains from Task Offshoring and Complementarity between Tasks^{*}

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Abstract

In this paper, I show that there are terms of trade effects associated with task offshoring. The offshoring source country may enjoy terms of trade gains associated with offshoring when the elasticity of substitution between tasks is low. These gains may stem from technology transfer from the source to the destination country or from the productivity improvement in the offshoring destination. Importantly, the offshoring destination country may suffer a total welfare loss due to the terms of trade effects. I illustrate that the source country will enjoy larger terms of trade gains when the elasticity of substitution between tasks is smaller and when the comparative advantage schedule has a steeper slope at the cutoff task.

JEL: F12, F15, F61.

Keywords: terms of Trade Gain, Offshoring, Elasticity of Substitution

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1 Introduction

It is a reasonable assumption that the elasticity of substitution between final goods (or varieties) is equal to or larger than unity. Such an assumption is widely used in trade models. For example, Dornbusch, Fischer and Samuelson (1977) assumes that the elasticity of substitution between goods is unity and Melitz (2003) assumes that the elasticity of substitution between varieties is greater than one.

However, when it comes to tasks that are needed to perform to produce a final good, the assumption becomes less relevant. Instead, there might be some complementarity between tasks. Take Smith's famous metaphor of the pin factory as an example. Pin production involve twelve distinct tasks: wire-drawing, wire-straightening, cutting, pointing, pin-head making, pin-head finishing, pin-head and pin assembly, finishing, washing, making boxes, box-filling and box-closing (Bianchi and Labory, 2011, p64). A certain quantity of each task is required and cannot be replaced by other tasks, even if the cost of performing the task rises.

The complementarity between tasks exists not only in the production of simple goods such as pins, it is also prevalent in more complex products such as engines or even software (Lanz, Miroudot and Nordås, 2012; Görlich, 2010). As a matter of fact, complementarity between tasks is the main reason for modularizing the production process (Grossbard-Shechtman and Clague, 2002). Given the complementary nature of tasks in production, models of tasks typically assume a certain degree of complementarily between tasks, for example in Kremer (1993) and Grossman and Rossi-Hansberg (2008).

This paper shows that task complementarity has important welfare implications for task offshoring. Specifically, I identify a new source of welfare gain for the source country associated with offshoring: terms of trade gains, when the elasticity of substitution between tasks is low (i.e. in the range of [0, 1)). The terms of trade gains may stem from the production technology transferred from the source to the destination country or from the productivity improvement in the destination country.¹

¹Technology transfer from the source to the destination country may occur when a firm from the developed country sets up a subsidiary in the developing country to perform offshored tasks. Li (2013) shows that in 2008 more than 60% of processing trade in China is conducted by wholly-foreign-owned firms

The intuition is simple. When the elasticity of substitution is low and when there is technology transfer from the source to the destination country, the cost (or value) of tasks exported by the destination country declines. To rebalance trade, the destination country needs to perform a larger range of tasks and the source country will consequently focus on tasks in which it has stronger comparative advantage. This concentration induces higher wages in the source country relative to the wages in the destination country and generates terms of trade gains.

Importantly, technology transfer from the source to the destination country may lead to a total welfare loss in the destination country. Although technology transfer directly reduces the cost of final goods and thus has positive impact on the welfare in the destination country, the terms of trade effects work against the destination country. The negative terms of trade effects may be so strong such that they outweigh the positive cost-saving effects. Since the destination country may suffer total welfare loss from its productivity improvement, I thus show the possibility of immiserizing growth in the context of task offshoring.²

Total welfare gains or losses for the destination country critically affect the incentives for the country to pursue the more efficient production technology. It is thus important to understand the factor that determines which of the two opposing effects, the terms of trade effects and the cost-saving effects, dominates. I show that the negative terms of trade effect is more likely to dominate when the comparative advantage curve has a larger slope (in absolute value) at the cutoff task.³

One way to reduce the slope of the comparative advantage curve so that the destination country has a stronger motivation to improve its productivity is to reduce the performing cost of *potential* tasks that the destination country is *not* performing. The destination country's R&D efforts in more advanced tasks, those performed in the source country but

and only around 10% is conducted by non-foreign-invested Chinese firms.

²Notice that immiserizing growth can happen in either the source country or the destination country, depending on which country experiences productivity improvement.

³The comparative advantage curve is defined similarly as in Dornbusch, Fischer and Samuelson (1977). In particular, if $c^*(s)$ denotes cost of performing task s in the destination country and c(s) in the source country, and if tasks are arranged such that $\frac{c^*(s)}{c(s)}$ is decreasing in the task index s, then the curve defined by $\frac{c^*(s)}{c(s)}$ is called "comparative advantage curve".

not in the destination country, should be encouraged since they will also encourage the destination country to utilize better technology in the tasks that it is currently performing.

Note that the above conclusions, especially the possibility of immizerizing growth, are obtained only in the task offshoring context when the elasticity of substitution between tasks is small. One obvious example is that immizerizing growth is ruled out in the Dornbusch, Fischer and Samuelson (1977) model when the elasticity of substitution is unity. The intuition is clear. When the elasticity of substitution is unity, the value share for each task is constant. When the destination country experiences productivity improvement in tasks that are already performed in the destination country, the productivity improvement reduces per-unit costs but does not affect the expenditure shares for these tasks. Thus trade remains balanced and the range of tasks performed in each country is unchanged. Both countries enjoy welfare gains due to the productivity improvement.⁴

To see the impacts of elasticity of substitution clearly, I extend the model to allow varying levels of elasticity of substitution. Specifically, I assume the production function for the final good is given by a constant elasticity of substitution (CES) function.⁵ I show that a smaller elasticity of substitution allocates a larger fraction of the welfare gain arising from destination productivity improvement to the source country. When the elasticity of substitution is below a threshold (a number smaller than one), technology transfer may, although not always, induce a total welfare loss in the destination country.

The paper has important implications to both offshoring theory and policies. In terms of theory, I show that, when the elasticity of substitution between tasks is zero, wages are determined by the pattern of specialization, which is in turn determined by productivity in the source and destination country. This indicates that lower labor cost in the developing country may be a result rather than a reason of offshoring. Offshoring models thus should be very cautious to assume exogenous lower wages in the developing countries as a reason of

⁴When the elasticity of substitution is unity, it is not *only* in the case, where productivity improvement in tasks that are already performed in the destination country (a case I called "inframarginal technology improvement" in later sections), that both countries are better off. They are both better off in the case of technology transfer (which I called "technology transfer efficiency improvement) as well.

⁵With this assumption, my model becomes a Dornbusch, Fischer and Samuelson (1977) framework with a CES production function. This is close to Opp (2010). Different from Opp (2010) where optimal tariff is discussed, I focus on the terms of trade effects from technology improvement.

offshoring. In terms of policy, the paper suggests that technology transfer from offshoring source country to the destination country, especially in tasks that are already offshored, might generate welfare gains to the source country. Thus, such technology transfer should be encouraged by the source country so that it can take advantage of the terms of trade effects.

The paper is organized as follows. In Section 2, I present a simple Dornbusch, Fischer and Samuelson (1977) model with a Leontief production function. I characterize the equilibrium and discuss the terms of trade effects. In Section 3, I generalize the model by adopting a CES production function and show that the magnitude of terms of trade effect is related to the elasticity of substitution. Section 4 concludes.

2 The Model

There are two countries (home and foreign), with respective labor endowments L and L^* . Asterisks are used throughout the paper to refer to the foreign country. In the context of offshoring, home is the offshoring source country and foreign is the destination country. There is one final good and the preferences over the final good are the same in the two countries. Assume, for example, home workers earn wage w and the price of the final good is p, then $\omega = w/p$ indicates the real wage and the utility level achieved at home.

2.1 Production of the Final Good

The final good is produced by performing a continuum of tasks index by $s \in [0, 1]$. Without loss of generality, we define the unit of each task such that for each unit of final good, one unit of each task must be performed.⁶ I.e., the production function of the final good is a Leontief function, $y = \min_{s \in [0,1]} \{x(s)\}$, where y is the quantity of the final good and x(s)is the units of each task s performed. Given this production function, the unit labor cost for the final good, A, is then

$$A = \int_0^1 a(s)ds,\tag{1}$$

where a(s) is the unit labor cost to perform each task s.

⁶Note that change of the definition of units for tasks does not affect the results qualitatively.

The unit labor costs can be decomposed as a(s) = tc(s), where t denotes home's aggregate technology level and c(s) denotes the task-specific cost. Similarly, the foreign unit labor cost is $a^*(s) = t^*c^*(s)$. I assume that home has more advanced technology than the foreign country, i.e. $t < t^*$.⁷

Home's comparative advantage differs across tasks. Without loss of generality, I assume that the tasks are ordered such that $\frac{c^*(s)}{c(s)}$ is decreasing in the task index, s. Thus home has stronger comparative advantage in tasks with lower index s. The curve defined by $\frac{c^*(s)}{c(s)}$ is called "the comparative advantage curve".

When final goods are freely traded but not tasks, final good prices across countries are equalized,

$$p = w \int_0^1 a(s) ds = w^* \int_0^1 a^*(s) ds,$$

where home wage is w and foreign wage is w^* . The real wage at home is thus 1/A and in the foreign country $1/A^*$.

2.2 Task Offshoring

We now allow tasks to be freely offshored. Assume that home's technology can be partly transferred to the foreign country, with a technology transfer efficiency e. I.e., the actual unit labor cost of performing task s in the foreign country is $et^*c^*(s)$, where $\frac{t}{t^*} \le e \le 1$. On the other hand, if a tasks is performed at home, since $t \le t^*$, the home unit labor cost is still tc(s).

Tasks will be performed in the country that costs less. The price of task s at home is wtc(s) and that in the foreign country is $w^*et^*c^*(s)$. Since $\frac{c^*(s)}{c(s)}$ is decreasing in the task index, s, in equilibrium there must exist a cutoff task \bar{s} such that

$$\frac{et^*c^*(\bar{s})}{tc(\bar{s})} = \frac{w}{w^*}.$$
(2)

Otherwise, either the home or the foreign country would have cheaper prices for all tasks, and it cannot be the equilibrium. Given this equilibrium condition, tasks with index $s \in [0, \bar{s}]$ will be performed at home and the remaining tasks performed in the foreign country.

⁷Home might have absolute advantage in all tasks. I.e. t so small such that $a(s) < a^*(s)$ for any task s.

The price of the final good with free offshoring is now

$$p^{o} = \int_{0}^{\bar{s}} tc(s)wds + \int_{\bar{s}}^{1} et^{*}c^{*}(s)w^{*}ds.$$
(3)

The home real wage is

$$\omega = \frac{w}{p^o} = \left(\int_0^{\bar{s}} tc(s)ds + \int_{\bar{s}}^1 et^*c^*(s)\frac{w^*}{w}ds\right)^{-1},\tag{4}$$

and that of the foreign country is

$$\omega^* = \frac{w^*}{p^o} = \left(\int_0^{\bar{s}} \frac{w}{w^*} tc(s) ds + \int_{\bar{s}}^1 et^* c^*(s) ds\right)^{-1}.$$
 (5)

Compared with the welfare achieved under autarky, because for tasks with index $s \in [\bar{s}, 1]$, $\frac{w^*et^*c^*(s)}{w} \leq tc(s)$, and for tasks with index $s \in [0, \bar{s}]$, $\frac{wtc(s)}{w^*} \leq et^*c^*(s) \leq t^*c^*(s)$, both countries are better off from free offshoring.

The trade balance equation completes the description of the equilibrium

$$L\int_{\bar{s}}^{1} et^{*}c^{*}(s)ds = L^{*}\int_{0}^{\bar{s}} tc(s)ds.$$
 (6)

Define $\theta(s) \equiv \frac{t \int_{s}^{s} c(j) dj}{et^* \int_{s}^{1} c^*(j) dj}$, the trade balance equation can be rewritten as $\theta(\bar{s}) = \frac{L}{L^*}$.

This simple equation conveys a lot of insights. Notice that $\theta(s)$ is an increasing function in s and it takes values from zero to infinite when $s \in [0, 1]$. Since the labor ratio is exogenous and takes a positive value, the cutoff \bar{s} is determined soly by the trade balance equation. The equilibrium value of \bar{s} is shown in Panel (a) of Figure 1. Once the equilibrium cutoff service \bar{s} is determined by the trade balance equation, the equilibrium relative wage is consequently determined by the equation (2), as shown in Panel (b) of Figure 1.

It is interesting and important to note that wages do not affect the pattern of trade. The intuition is that, an increase of home wage will not only increase the value of exported tasks by home, it will also increase the demanded value of imported tasks from the foreign country, leaving trade balanced. The only determinant of the trade pattern is the production technologies in the two countries. Once the trade pattern and the cutoff task \bar{s} are determined, the relative wage is consequently determined.

Proposition 1. With Leontieff production function of final good, under free offshoring, wage changes will not affect the trade balance.



Figure 1: Service Equilibrium with One Final Good

Proposition 1 is worth emphasizing for two reasons. First, it is in sharp contrast with the results when the production function is Cobb-Douglas, as in Dornbusch, Fischer and Samuelson (1977). In their case, wages do affect the trade balance while production technologies play no role, since the expenditure share of each task is exogenously given. In my case, wages play no role while production technologies are the key to determine the trade balance.

Second, intuition and some partial equilibrium offshoring models suggest that one of the major driving force of offshoring is that developed countries take advantage of lower labor wage in the developing countries. Proposition 1 suggests that, in general equilibrium, the lower labor cost in the developing country may be a result rather than a reason of offshoring.

2.3 Terms of Trade Effects

As relative wage affects the welfare level, factors that cause terms of trade changes would also affect the welfare level. In this section, we focus on the factors that may cause terms of trade changes.

Equilibrium conditions, Equations (2) and (6), indicate that production technologies in the two countries affect the trade balance and consequently the relative wages. We will focus on the production technology change in the foreign country and study how such changes will affect the welfare levels in the two countries.

There are potentially three types of changes of production technology in the foreign country, 1) the technology transfer efficiency from the home to the foreign improves, i.e. e becomes smaller, 2) an overall technology improvement in the foreign country, i.e. t^* reduces, and 3) technology improvements in tasks originally performed in the foreign country, i.e. t^* reduces but only for $s \in [\bar{s}, 1]$. The first two types of changes are isomorphic and I shall refer to any such changes as a "technology transfer efficiency improvement".⁸ The third type of changes applies to the set of tasks that are originally performed in the foreign country. I shall refer to these changes as "inframarginal technology improvement".

2.3.1 Technology Transfer Efficiency Improvement

In this subsection, I consider impacts of a technology transfer efficiency improvement. Following Equation (6), Appendix 1 shows that a technology transfer efficiency improvement will induce a larger range of tasks performed in the foreign country. I.e.,

$$\frac{d\bar{s}}{de} = \left(e\left(\frac{c(\bar{s})}{\int_0^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^1 c^*(s)ds}\right)\right)^{-1} > 0.$$

The intuition is that more efficient technology transfer lower unit labor requirement in the foreign country. It consequently reduces the total value of tasks exported by the foreign country and thus the home country must purchase a larger range of tasks to rebalance the trade. This is shown in Figure 2.

The welfare change in the two countries critically hinges on the change of the relative wage, w/w^* . By Equation (2), we have

$$\frac{d(w/w^*)}{de} = \frac{t^*}{t} \left(\frac{c^*(\bar{s})}{c(\bar{s})} + e \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \frac{d\bar{s}}{de} \right).$$
(7)

When technology transfer efficiency improves (de < 0), there are two effects on the relative wage. The first is a negative effect that is directly due to lower foreign cost at the cutoff

⁸Notice that technology transfer efficiency improvement has the same effects as a common technology improvement in tasks $s \in [\bar{s}', 1]$, where \bar{s}' is the cutoff task in the *new* equilibrium after the technology improvement and $\bar{s}' < \bar{s}$.



Figure 2: Equilibrium Change with Technology Transfer Efficiency Improvement

task, $\frac{c^*(\bar{s})}{c(\bar{s})} > 0$. The second effect is positive, due to the lower cutoff task, $e\frac{d\bar{s}}{de}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} < 0$. Which of these two effects dominates determines the overall sign of the terms of trade effect.

In general, which effect dominates depends on the functional form of $\frac{c^*(s)}{c(s)}$ at the cutoff task \bar{s} . In fact, the first effect always dominates for some simple functional forms of $\frac{c^*(s)}{c(s)}$.⁹ However, it is not rare that the second effect may dominate. In fact, Lemma 1 establishes that at any point $\bar{s} \in [0, 1]$ for any given function, $\frac{c^*(s)}{c(s)}$, after a small local adjustment of the function around \bar{s} , the second effect can always dominate.

Lemma 1. Given a point $\bar{s} \in [0, 1]$ and a function, $c^*(s)/c(s)$, which is decreasing, continuous and differentiable. A new function of $c^*(s)/c(s)$ can be constructed such that it is the same as the original given function except at a infinitely small neighborhood around \bar{s} , and that

$$\frac{c^*(\bar{s})}{c(\bar{s})} + e \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \frac{d\bar{s}}{de} < 0.$$

The newly constructed function is also decreasing, continuous and differentiable.

Proof. See Appendix 2.

⁹For example, the first effect always dominates if $\frac{c^*(s)}{c(s)}$ takes a power function (e.g. $\frac{c^*(s)}{c(s)} = s^{-\lambda}$, where $\lambda > 0$), an exponential function (e.g. $\frac{c^*(s)}{c(s)} = \lambda^{1-s}$, where $\lambda > 1$), or a linear function (e.g. $\frac{c^*(s)}{c(s)} = \lambda_1(1s) + \lambda_2$, where $\lambda_1 > 0$, $\lambda_2 > 0$).

The intuition of the proof is as follows. Notice that given \bar{s} , c(s) for $s \in [0, 1]$ and $c^*(s)$ for $s \in [\bar{s}, 1]$, $\frac{d\bar{s}}{de}$ is positive and finite. In other words, the extent of the reduction of cutoff task due to the improvement of technology transfer efficiency has nothing to do with the functional form of $c^*(s)$ for $s \in [0, \bar{s})$. However, $\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}$ is determined by the functional form of $c^*(s)$ for $s \to \bar{s}$, when c(s) for $s \in [0, 1]$ is given. Thus, arbitrarily increase the slope of $c^*(s)$ at $s \to \bar{s}$, the derivative, $\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}$, can achieve any value in $[-\infty, 0]$.

Given that the home relative wage may increase or decrease due to technology transfer efficiency improvements, the following proposition follows:

Proposition 2. The home relative wage may increase or decrease when the technology transfer efficiency improves. Define a finite number $n \equiv \left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / \left(e\frac{d\bar{s}}{de}\right) > 0$ at $s = \bar{s}$. If $d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / d\bar{s} \in [-n, 0]$ then home relative wage decreases, and if $d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / d\bar{s} \in [-\infty, -n]$, home relative wage increases.

Proof. The proof immediately follows Equation (7). \Box

We can now study the welfare changes due to technology transfer efficiency improvement. For the home country, notice that $\frac{et^*c^*(\bar{s})}{tc(\bar{s})} = \frac{w}{w^*}$ in equilibrium, Equation (4) can be rewritten as

$$\omega = \left(\int_0^{\bar{s}} tc(s)ds + \frac{tc(\bar{s})}{t^*c^*(\bar{s})} \int_{\bar{s}}^1 t^*c^*(s)ds\right)^{-1}.$$
(8)

Taking derivative respect to the cutoff task \bar{s} , we have that

$$\frac{d\omega}{d\bar{s}} = -\omega^2 t \int_{\bar{s}}^1 c^*(s) ds \frac{d\left(\frac{c(\bar{s})}{c^*(\bar{s})}\right)}{d\bar{s}} < 0.$$
(9)

We thus have shown that when technology transfer efficiency improves, the cutoff task \bar{s} decreases. Consequently, home welfare unambiguously increases. The intuition is that, as technology transfer efficiency improves, the unit labor requirement for each task performed in the foreign country decreases. This tends to reduce the price of the final good and thus increase the home welfare. In the same time, as the technology transfer efficiency increases, the home relative wage is subject to the two offsetting effects as shown by Equation (7). The negative effect of technology transfer efficiency on home relative wage precisely cancels the home welfare gain due to foreign unit labor requirement reduction. This can be seen

by the cancelation of technology transfer efficiency e in the second term in the bracket in Equation (8). The leftover positive effect of technology transfer efficiency on home relative wage thus induces unambiguous higher home welfare.

The welfare gain at home due to the improved terms of trade can be big, since the terms of trade change affects the welfare not through marginal tasks, but through inframarginal tasks. This can be seen in Equation (9), where the home relative wage is multiplied by an integral of foreign performed tasks, $s \in [\bar{s}, 1]$. This indicates that the terms of trade are of great importance for country's welfare. Any offshoring theory that ignores the terms of trade change might also ignore a important channel of welfare changes.

Compared to the home welfare, the foreign country's welfare is a bit more complicated. To gain some intuition, we can write foreign real wage as

$$\omega^* = \left(et^*\left(\frac{c^*(\bar{s})}{c(\bar{s})}\int_0^{\bar{s}} c(s)ds + \int_{\bar{s}}^1 c^*(s)ds\right)\right)^{-1}$$

Taking derivative of ω^* respect to e, we have

$$\frac{d\omega^*}{de} = -\omega^{*2} \left(\frac{t^* c^*(\bar{s})}{c(\bar{s})} \int_0^{\bar{s}} c(s) ds + t^* \int_{\bar{s}}^1 c^*(s) ds + et^* \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \frac{d\bar{s}}{de} \int_0^{\bar{s}} c(s) ds \right).$$

Technology transfer efficiency improvement has two opposite effects on the foreign welfare. First, a lower e reduces the costs of performing tasks in the foreign country. At the same time, the lower e also directly reduces home relative wage, which in turn lower the costs of performing tasks at home. Both these effects increases foreign welfare, shown by the first and second term in the bracket. Second, a lower e would induce a lower \bar{s} , which tends to increase home relative wage. Such an increase of home relative wage applies to all tasks performed at home, $s \in [0, \bar{s}]$, and tends to reduce foreign welfare. This effect is captured by the third term in the bracket.

The latter negative impact on foreign welfare may or may not dominate the former positive impact, depending on the slope of the comparative advantage curve at \bar{s} . When the slope is small, the former effect dominates and the foreign welfare increases. When the slope is large, the latter effect dominates and the foreign loses. Formally, we have,

Proposition 3. Home country welfare always increases as the technology transfer efficiency improves. The foreign welfare may increase or decrease depending on the slope of the comparative advantage curve at the cutoff task. Given finite numbers of n and m, which are defined as $0 < n \equiv \left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / \left(e\frac{d\bar{s}}{de}\right) < m \equiv \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{tL^*}{t^*eL}\right) / \left(e\frac{d\bar{s}}{de}\right)$, if $d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / d\bar{s}$ is in the range: 1) [-n, 0], then home relative wage decreases and foreign welfare increases; 2) [-m, -n], then home relative wage increases, but foreign welfare still increases; 3) $[-\infty, -m]$, then home relative wage increases and foreign welfare still decrease.

Proof. See Appendix 3.

Proposition 3 highlights that the terms of trade effects play important roles in the welfare gains from offshoring. If offshoring is associated with technology transfer from the developed country to the developing country, an improvement in technology transfer efficiency always benefits the developed country but may actually hurt the developing country.

Our results that foreign may be worse off due to technology transfer efficiency improvement is in sharp contrast to the results when tasks are more substitutable. For example, in Dornbusch, Fischer and Samuelson (1977) where the production function is of Cobb-Douglas form, a technology transfer efficiency improvement will always induce higher foreign relative wage and thus higher welfare in the foreign country.¹⁰

Having shown that the foreign welfare may be lower with a more efficient technology transfer, it is worth noting that compared to autarky (no offshoring), offshoring is always beneficial for both countries. In the case that we have large foreign welfare loss induced by technology transfer efficiency improvement, it is also the case that the welfare gain from offshoring, compared with autarky, is large for the foreign country. Thus, the welfare loss from technology transfer efficiency improvement will never be large enough to completely offset the welfare gain from offshoring.

In the beginning of section 2.3, we have emphasized that the technology transfer efficiency improvement, i.e. a lower e, has identical impacts as an overall production technology improvement in the foreign country, i.e. a lower t^* . Thus, the above analysis shows us one possible occasion of "immizerizing growth". In the context of offshoring when tasks

¹⁰Of course, in Dornbusch, Fischer and Samuelson (1977), they discussed neither offshoring nor technology transfer efficiency improvement. However, the point can be easily seen by replacing the Leontieff production function in my model by a Cobb-Douglas function. It is also shown in Section 3.

are complimentary, a production technology improvement in one country (no matter the sourcing country or the destination country) may induce too low relative wage for the country, and thus causes lower welfare level in that country.

2.3.2 Inframarginal Technology Improvement

Section 2.3.1 discusses the terms of trade effect of an improvement in technology transfer efficiency (i.e. lower e), which is equivalent to a universal production technology improvement in the foreign country (i.e. lower t^*). In fact, as pointed out in Footnote 8, a common technology improvement in tasks $s \in [\bar{s}', 1]$, where \bar{s}' is the cutoff task in the new equilibrium after the technology improvement, will have the same effects as described in the above subsection. The similarity among these cases is that the change in foreign technology affected the new marginal task, \bar{s}' , as well as all inframarginal tasks, $s \in [\bar{s}, 1]$. How will the effects change if a technology improvement only affects the inframarginal tasks? I will investigate this question in this subsection.

We start from an equilibrium in which cutoff task is \bar{s} . For simplicity of notation, assume that $t^* = \bar{t}^*$ for $s \in [0, \bar{s})$, where \bar{t}^* is a fixed number, and $t^* = \tilde{t}^*$ for $s \in [\bar{s}, 1]$, where \tilde{t}^* can change. Also assume that $\tilde{t}^* = \bar{t}^*$ in the original equilibrium.

In this original equilibrium, the trade balance equation is given by Equation (6), with \tilde{t}^* replacing t^* : $L \int_{\bar{s}}^1 e\tilde{t}^*c^*(s)ds = L^* \int_0^{\bar{s}} tc(s)ds$. As inframarginal technology improvement happens, \tilde{t}^* decreases and the value of the exported tasks by the foreign country becomes lower. Such a change is shown in Panel (a) of Figure 3. To rebalance the trade, foreign expands its range of tasks performed. The equilibrium cutoff task is reduced from \bar{s} to \bar{s}' . The new trade balance equation is given by

$$L\left(\int_{\bar{s}'}^{\bar{s}} e\bar{t}^* c^*(s) ds + \int_{\bar{s}}^{1} e\tilde{t}^* c^*(s) ds\right) = L^* \int_0^{\bar{s}'} tc(s) ds.$$
(10)

The function $\theta(s)$ is now redefined as

$$\theta(s) = \frac{t \int_{0}^{\bar{s}} c(j) dj}{\int_{\bar{s}'}^{\bar{s}} e\bar{t}^{*} c^{*}(j) dj + \int_{\bar{s}}^{1} e\tilde{t}^{*} c^{*}(j) dj}$$

and the trade balance equation can be rewritten as $\theta(\bar{s}') = \frac{L}{L^*}$. It is obvious from this new definition that $\theta(s)$ will pivots up on the origin when \tilde{t}^* decreases, but with a relatively



(a) Inframarginal Technology Improvement

(b) Trade Balance with Inframarginal Technology Improvement

Figure 3: Equilibrium Change with Inframarginal Technology Improvement in the Foreign

smaller magnitude, compared to the case when the productivity improvement applies to all task $s \in [0, 1]$. This change is shown in Panel (b) of Figure 3.

As the new equilibrium becomes $\bar{s}' < \bar{s}$, the relative wage equation becomes

$$\frac{w}{w^*} = \frac{e\bar{t}^*c^*(\bar{s}')}{tc(\bar{s}')}.$$
(11)

The equilibrium relative wage now will be changed only through the change of the cutoff task \bar{s}' , but not $\frac{e\bar{t}^*}{t}$, as it is fixed.

Our object is to study how reduction of \tilde{t}^* will affect the equilibrium cutoff task, \bar{s}' , the relative wages and the welfare levels. Given the trade balance equation (10), we can show that $\frac{d\bar{s}'}{d\bar{t}^*} > 0$ (see Appendix 4). Consequently, the change of home relative wage is given by

$$\frac{d(w/w^*)}{d\tilde{t}^*} = \frac{e\bar{t}^*}{t} \frac{d\left(\frac{c^*(\bar{s}')}{c(\bar{s}')}\right)}{d\bar{s}'} \frac{d\bar{s}'}{d\tilde{t}^*} < 0.$$
(12)

Since inframarginal technology improvement lowers equilibrium cutoff task, \bar{s}' , without changing the production technology at the new cutoff, the home relative wage always gets higher.

This is in contrast to the case of technology transfer efficiency improvement, where there are forces to decrease the home relative wage. As inframarginal technology improvement generates larger home relative wage than technology transfer efficiency improvement, it is to be expected that home country gains more while the foreign country gains less from inframarginal technology improvement than from technology transfer efficiency improvement. Indeed, we have the following proposition,

Proposition 4. Inframarginal technology improvement in the foreign country always increase the home relative wage and home welfare. It may or may not increase foreign welfare. Define $m' \equiv \frac{c(\bar{s}')}{\left(\int_{0}^{\bar{s}'} c(s)ds\right)^2} \left(\int_{\bar{s}'}^{\bar{s}} c^*(s)ds + \frac{\tilde{t}^*}{t^*} \int_{\bar{s}}^{1} c^*(s)ds\right) + \frac{c^*(\bar{s}')}{\int_{0}^{\bar{s}'} c(s)ds} > 0$. If $d\left(\frac{c^*(\bar{s}')}{c(\bar{s}')}\right)/d\bar{s}' \in [-m', 0]$, then foreign welfare will increase with foreign inframarginal technology improvement and if $d\left(\frac{c^*(\bar{s}')}{c(\bar{s}')}\right)/d\bar{s}' \in [-\infty, -m']$, then foreign welfare will de-

Proof. See Appendix 5.

crease.

Notice that, letting $\bar{s} = \bar{s}'$, $\bar{t}^* = \tilde{t}^* = t^*$, we can show that m' < m. This indicates that we need a smaller slope of the comparative advantage curve in order to generate total welfare loss for the foreign country in the case of inframarginal technology improvement than in the case of technology transfer efficiency improvement. It confirms that inframarginal technology improvement is less favorable to the foreign country than a same magnitude change of technology transfer efficiency improvement.

To sum up Section 2, we have shown that, when the production of final good takes a Leontieff function, offshoring is associated with terms of trade gain to the source country. Such a terms of trade gain may stem from technology transfer from the source country to the destination country or from the productivity improvement in the destination country. Moreover, the source country will enjoy larger terms of trade gain if the slope of the comparative advantage curve at the cutoff task is larger. On the other hand, technology transfer or productivity improvement in the destination country may, although not always, result in total welfare loss to the destination country, presenting a case of "*immizerizing growth*" in the context of offshoring.

3 Extension: CES Production Function

In this section I shall extend the model by adopting a more general production function: the CES function.¹¹ The object of this section is to show that, when there is technology improvement in the destination country, home enjoys larger terms of trade gains if the elasticity of substitution is smaller. Moreover, the destination country may suffer immizerizing growth only when the elasticity of substitution is less than one.¹²

3.1 Production of the Final Good

The main setup of the model is as described in Section 2, but now the production function of the final good takes a CES form: $y = \left(\int_0^1 b(s)x(s)^{\frac{\sigma-1}{\sigma}}ds\right)^{\frac{\sigma}{\sigma-1}}$, where b(s) is the share parameter for task s.¹³ The parameter σ is the elasticity of substitution between tasks and it can take any value in $[0, \infty]$.

Given CES production function, the price of the final good is

$$p = \left(\int_0^1 b(s)^{\sigma} p(s)^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}},$$

where p(s) is the unit performing cost of task s. The unit performing cost is wtc(s) if the task is performed at home and $w^*t^*c^*(s)$ if performed in the foreign country without offshoring. The units of tasks needed to perform in order to produce y units of the final good is $x(s) = yb(s)^{\sigma} \left(\frac{p(s)}{p}\right)^{-\sigma}$, so the cost share of task s is then

$$\delta(s) = \frac{p(s)x(s)}{py} = b(s)^{\sigma} \left(\frac{p(s)}{p}\right)^{1-\sigma}.$$
(13)

With free trade of final good but not tasks, the final good price across countries are

¹¹Notice that when I assume a CES production function and allow the elasticity of substitution to take any value in $[0, \infty]$, the model can be interpreted as a final good trade model as well. The Dornbusch, Fischer and Samuelson (1977) is a special case with elasticity of substitution equal to one and the model in Section 2 is a special case with the elasticity equal to zero.

¹²More specifically, in the case of technology transfer efficiency improvement, the destination country may suffer immizerizing growth when the elasticity of substitution is smaller than a threshold that is less than one. In the inframarginal technology improvement case, it may happen when the elasticity of substitution is smaller than one.

¹³Notice that b(s) is the cost share of task s when $\sigma = 1$.

equalized as

$$p = wt \left(\int_0^1 b(s)^{\sigma} c(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}} = w^* t^* \left(\int_0^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}}$$

The real wage at home and in the foreign country are respectively

$$\omega = \left(t \left(\int_0^1 b(s)^\sigma c(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}} \right)^{-1}$$

and

$$\omega^* = \left(t^* \left(\int_0^1 b(s)^\sigma c(s)^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}\right)^{-1}.$$

3.2 Offshoring

When tasks are freely offshored, the unit cost of performing task s at home is wtc(s) and $ew^*t^*c^*(s)$ in the foreign country. The two countries specialize in tasks that they have comparative advantage. The pattern of specialization is still determined by Equation (2). I.e., tasks with index $s \in [0, \bar{s}]$ will be performed at home and otherwise in the foreign country.

With free offshoring, the price of the final good becomes

$$p^{o} = \left((wt)^{1-\sigma} \int_{0}^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds + (ew^{*}t^{*})^{1-\sigma} \int_{\bar{s}}^{1} b(s)^{\sigma} c^{*}(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}}.$$
 (14)

The two countries' real wages are $\omega = w/p^o$ and $\omega^* = w^*/p^o$, respectively. Again, because of the specialization pattern, $wtc(s) \leq ew^*t^*c^*(s) \leq w^*t^*c^*(s)$ for $s \in [0, \bar{s}]$ and $ew^*t^*c^*(s) \leq wtc(s)$ for $s \in [\bar{s}, 1]$. The two countries are both better off compared to the case of no offshoring.

The trade balance equation is given by $wL\int_{\bar{s}}^{1}\delta(s)ds = w^{*}L^{*}\int_{0}^{\bar{s}}\delta(s)ds$, where

$$\delta(s) = \begin{cases} \left(\frac{etw}{p^{o}}\right)^{1-\sigma} b(s)^{\sigma} c(s)^{1-\sigma} & s \in [0,\bar{s}]\\ \left(\frac{et^*w^*}{p^{o}}\right)^{1-\sigma} b(s)^{\sigma} c^*(s)^{1-\sigma} & s \in [\bar{s},1] \end{cases}$$

It can be rewritten as

$$\frac{w}{w^*} = \left(\frac{L^*}{L}\right)^{\frac{1}{\sigma}} \left(\frac{t}{et^*}\right)^{\frac{1-\sigma}{\sigma}} \left(\frac{\int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds}{\int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds}\right)^{\frac{1}{\sigma}}.$$
(15)



Figure 4: Offshoring Equilibrium under CES Production Function

Define $\vartheta(s) \equiv \left(\frac{L^*}{L}\right)^{\frac{1}{\sigma}} \left(\frac{t}{et^*}\right)^{\frac{1-\sigma}{\sigma}} \left(\frac{\int_0^s b(s)^{\sigma} c(s)^{1-\sigma} ds}{\int_s^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds}\right)^{\frac{1}{\sigma}}$. For any given $\sigma \neq 0$, $\vartheta(s)$ is an increasing function in s, taking values from zero to infinite when s goes from zero to unit. Thus Equation (15) together with Equation (2) determines the equilibrium cutoff task \bar{s} and the relative wage w/w^* . This is shown in Figure 4, where Equation (15) is represented by the trade balance curve (TB curve) and Equation (2) by the cutoff task curve (CO curve). It is interesting to note that, when $\sigma = 0$, the curve $w/w^* = \vartheta(s)$ becomes a vertical line in Figure 4. This vertical line goes through \bar{s} which is such that $\frac{L^*}{L} \frac{\int_0^{\bar{s}} c(s) ds}{\int_{\bar{s}}^1 c^*(s) ds} = 1$, as shown in Panel (b) of Figure 1.

3.3 Terms of Trade Effect with CES Production Function

This section analyzes how terms of trade effects from offshoring are affected by the elasticity of substitution. For exposition simplicity, we focus on the case of $\sigma \in (0, \infty)$ and ignore the perfect substitution case in which $\sigma = \infty$.

3.3.1 Technology Transfer Efficiency Improvement

We start with technology transfer efficiency improvement, i.e. a reduction of e. The impacts of technology transfer efficiency improvement on the equilibrium can be best viewed in Figure 4.

As technology transfer efficiency improves, the CO curve shifts down. The TB curve



Figure 5: Equilibrium Changes with Technology Transfer Efficiency Improvement, CES Case

would also be affected but may pivot up or down depending on the elasticity of substitution. If the elasticity of substitution is greater than one, the TB curve pivots down as shown in Panel (a) of Figure 5. The larger is the elasticity of substitution, the larger is the magnitude of the shift. When $\sigma = 1$, the TB curve will not be affected. When σ gets closer to ∞ , the magnitude of the TB curve shift gets closer to that of the CO curve shift at the cutoff task.

When the elasticity of substitution is less than one, the TB curve pivots up as e gets lower. The shifts of both the CO curve and the TB curve reduce the cutoff task \bar{s} , but their impact on the relative wage is ambiguous. The downward shift of the CO curve tends to reduce the home relative wage, but the upward shift of the TB curve tends to increase it. Panel (b) of Figure 5 shows a case where the shift of TB curve dominates that of the CO curve such that the home relative wage increases. Formally, we have the following proposition,

Proposition 5. An improvement of technology transfer efficiency always induces a lower cutoff task, when $\sigma \in (0, \infty)$. When $\sigma \in [1, \infty)$, $d\left(\frac{w}{w^*}\right)/de > 0$, so an improvement of technology transfer efficiency induces lower home relative wage. When $\sigma \in (0, 1)$, the home

relative wage may increase or decrease depending on the slope of the $c^*(\bar{s})/c(\bar{s})$ curve at \bar{s} . It is as described as in Proposition 2 in this case.

Proof. See Appendix 6.

Similar as in Section 2, when $\sigma \in (0, 1)$ and the slope of the $\frac{c^*(\bar{s})}{c(\bar{s})}$ curve at \bar{s} is large, home relative wage may increase with the improvement of technology transfer efficiency. The intuition is the same. When $\sigma \in (0, 1)$, the improvement of technology transfer efficiency will induce the TB curve to pivot up. If this is associated with steep CO curve at \bar{s} , then a small shift of the TB curve would induce a large increase of the home relative wage.

Proposition 5 shows that the terms of trade effects associated with offshoring are tightly related with the elasticity of substitution and the slope of the comparative advantage curve. Higher elasticity of substitution tends to induce a lower home relative wage when there is technology transfer efficiency improvement. On the other hand, home relative wage is more likely to increase with technology transfer efficiency improvement when the comparative advantage curve has a larger slope. Thus the terms of trade gains from offshoring for the home is larger when the elasticity of substitution between tasks is smaller and when the comparative advantage curve is steeper at the cutoff task.

Turning to the overall welfare changes in the two countries. For the home, taking derivative of its real wage respect \bar{s} , we have

$$\frac{d\omega}{d\bar{s}} = -\omega^{2-\sigma}t^{1-\sigma}\left(\frac{c(\bar{s})}{c^*(\bar{s})}\right)^{-\sigma}\int_{\bar{s}}^1 b(s)^{\sigma}c^*(s)^{1-\sigma}ds\frac{d\left(\frac{c(\bar{s})}{c^*(\bar{s})}\right)}{d\bar{s}} < 0.$$

Thus as lower e lowers \bar{s} , home always gains from more efficient technology transfer. We have shown earlier that home relative wage may decrease in many cases with technology transfer efficiency improvement. This is especially true when the elasticity of substitution is in the range $\sigma \in [1, \infty)$. However, the lower home relative wage will never induce welfare loss for the home country. The intuition is the same as in Section 2. The direct reduction of home relative wage due to lower e is completely offset by the increased productivity in the foreign country. On the other hand, lower e induced concentration of tasks for the home country (lower \bar{s}), which increases home relative wage. This causes the net welfare effect in favor of home country.

For the foreign country, taking derivative of the real wage respect to e, we have that,

$$\frac{d\omega^*}{de} = -\omega^{*2-\sigma} \int_0^{\bar{s}} b(s)^{\sigma} \left(t^* c(s)\right)^{1-\sigma} ds \left(\frac{ec^*(\bar{s})}{c(\bar{s})}\right)^{-\sigma} \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*} + e\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\frac{d\bar{s}}{de}\right).$$
(16)

Further algebra (see Appendix 7) shows that the sign of $\frac{d\omega^*}{de}$ depends on two factors. First, the magnitude of σ . Define $\beta = \left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*}\right) < 1$. When $\beta \leq \sigma$, $\frac{d\omega^*}{de} < 0$ always. Thus, when the elasticity of substitution is larger than the threshold level β , foreign always gain from technology transfer efficiency improvement.

When $0 \leq \sigma < \beta$, the sign of $\frac{d\omega^*}{de}$ also depends on the slope of the comparative advantage curve. If the slope of the curve is large enough, then $\frac{d\omega^*}{de} > 0$ and the foreign country may be worse off from technology transfer efficiency improvement. When the slope of the curve is small, $\frac{d\omega^*}{de} < 0$ and the foreign country can still be better off. The following proposition follows.

Proposition 6. With CES production function, home country is always better off from an improvement of technology transfer efficiency. For the foreign country, when $\sigma \in [\beta, \infty)$, where $\beta = \left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*}\right) < 1$, it will also be better off. When $\sigma \in [0, \beta)$, the foreign welfare change follows the statement in Proposition 3.

Proof. See Appendix 7.

3.3.2 Inframarginal Technology Improvement

I now turn to the case of inframarginal technology improvement. Similar to Section 2, we expect that inframarginal technology improvement is less favorable to the foreign country than technology transfer efficiency improvement. Thus we expect larger gains to the home country and less gains for the foreign in the case of inframarginal technology improvement than in the case of technology transfer efficiency improvement.

We again start from an original equilibrium in which cutoff task is \bar{s} . Assume that $t^* = \bar{t}^*$ for $s \in [0, \bar{s}]$, where \bar{t}^* is a fixed number, and $t^* = \tilde{t}^*$ for $s \in (s, 1]$, where \tilde{t}^* can

change. We assume that $\tilde{t}^* = \bar{t}^*$ in the original equilibrium.

The trade balance equation at the original equilibrium is given by Equation (15), with \tilde{t}^* replacing t^* :

$$\frac{w}{w^*} = \left(\frac{L^*}{L}\right)^{\frac{1}{\sigma}} \left(\frac{t}{e\tilde{t}^*}\right)^{\frac{1-\sigma}{\sigma}} \left(\frac{\int_0^{\overline{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds}{\int_{\overline{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds}\right)^{\frac{1}{\sigma}}.$$
(17)

As inframarginal technology improves, \tilde{t}^* decreases. This induces both the CO curve and TB curve in Figure 4 to shift.

The CO curve is shifted down for $s \in (\bar{s}, 1]$ as inframarginal technology improvement only affects the tasks already performed in the foreign country. The impact of lower \tilde{t}^* on the TB curve is the same as a lower e. I.e., if the elasticity of substitution is equal to one, the TB curve will not be affected. If it is greater than one, the TB curve will pivot down on the origin, but with a magnitude less than that of the shift of the CO curve at the cutoff task. In both of these cases, the cutoff task will not change. The equilibrium relative wage is pinned down soly by the trade balance equation. These changes are shown in Panel (a) of Figure 6.¹⁴

If the elasticity of substitution is less than unit, the TB curve will pivot up as shown in Panel (b) of Figure 6. The cutoff task becomes lower, $\bar{s}' < \bar{s}$, and the home relative wage rises. At the new equilibrium \bar{s}' , the trade balance equation becomes

$$\frac{w}{w^*} = \left(\frac{L^*}{L}\right)^{\frac{1}{\sigma}} \left(\frac{t}{e}\right)^{\frac{1-\sigma}{\sigma}} \left(\frac{\int_0^{\bar{s}'} b(s)^{\sigma} c(s)^{1-\sigma} ds}{\bar{t}^{*1-\sigma} \int_{\bar{s}'}^{\bar{s}} b(s)^{\sigma} c^*(s)^{1-\sigma} ds + \tilde{t}^{*1-\sigma} \int_{\bar{s}}^{1} b(s)^{\sigma} c^*(s)^{1-\sigma} ds}\right)^{\frac{1}{\sigma}}.$$
 (18)

The relative wage also satisfies Equation (11). These two equations determine the new equilibrium.

Since the equilibrium changes are different for inframarginal technology improvement when the elasticity of substitution are different, we need to differentiate the cases of σ to discuss the welfare changes.

In the case that $\sigma > 1$, Proposition 7 shows that both countries are better off from the inframarginal technology improvement. Thus, even though inframarginal technology

¹⁴It is interesting to note that, unlike to all other cases we have discussed, when $\sigma \geq 1$, inframarginal technology improvement will not affect the specialization pattern, as the cutoff task \bar{s} is not changed.



Figure 6: Equilibrium Change (CES) with Inframarginal Technology Improvement in the Foreign

improvement is less favorable to the foreign country than technology transfer efficiency improvement, high elasticity of substitution ensures a sufficient amount of welfare gain from the technology improvement distributed to the foreign country, so that the foreign country is better off.

In the case that $\sigma \in (0,1)$, from Equation (18) and (11), simple algebra reveals that that $\frac{d\bar{s}'}{d\bar{t}^*} > 0$. By Equation (11), the change of relative wage is given by Equation (12). Since inframarginal technology improvement does not *directly* affect the home relative wage, except through the channel of cutoff task, \bar{s}' , similarly as in Section 2, the home relative wage thus always increases.

Also similar to Section 2, Proposition 7 indicates that, when $\sigma \in (0, 1)$, inframarginal technology improvement always increase home real wage, but not necessarily for the foreign real wage. The inframarginal technology improvement has two offsetting impacts on the foreign welfare. First, it unambiguously increases the home relative wage and leads to welfare loss to the foreign country. Second, it reduces the cost of performing tasks in the foreign and reduces the price of the final good. This contribute positively to the foreign welfare. The net change of welfare in the foreign country again depends on the slope of the comparative advantage curve at the cutoff task. If the comparative advantage curve at $s = \bar{s}'$ is steep enough, the foreign may be worse off, and otherwise better off.

Proposition 7. When elasticity of substitution is in the range $(1, \infty)$, inframarginal technology improvement always induces lower home relative wage, and both home and foreign country are better off. On the other hand, when elasticity of substitution is in the range (0, 1), inframarginal technology improvement always increases home relative wage and home welfare. Foreign welfare may or may not increase depending on the slope of the comparative advantage curve, in a way similar as in Proposition 4.

Proof. See Appendix 8.

To sum up Section 3, I have shown that, when there is technology improvement in the destination country, home enjoys larger terms of trade gains if the elasticity of substitution is smaller. Importantly, home relative wage might increase only when the elasticity of substitution between tasks is less than one. I also show that technology transfer efficiency improvement in the destination country may result in welfare loss to the destination country, only when the elasticity of substitution is smaller than a smaller-than-one threshold. Thus, "*immizerizing growth*" is more likely to be present in the context of offshoring rather than in the context of final good trade, as the elasticity of substitution between tasks is usually smaller than that between final goods.

4 Conclusion

This paper identifies a new source of welfare gain to the source country associated with offshoring: terms of trade gains, when the elasticity of substitution between tasks is low. The terms of trade gains may stem from the production technology transfer from the source to the destination country or from the productivity improvement in the destination country. I show that the size of the terms of trade gains in the source country is related to the elasticity of substitution and the slope of the comparative advantage curve at the cutoff task. More specifically, the source country will enjoy larger terms of trade gains when the elasticity of substitution between tasks is smaller and when the comparative advantage schedule has a larger slope.

Importantly, the terms of trade gains in the source country may be associated with a total welfare loss in the destination country. Since the destination country may suffer this "*immizerizing growth*" only when the elasticity of substitution is sufficiently small (typically smaller than one), I thus show that "*immizerizing growth*" is more likely to be present in the context of offshoring rather than in the context of final good trade, as the elasticity of substitution between tasks is usually smaller than that between final goods.

Since the destination country is less likely to experience the "*immizerizing growth*" if the slope of the comparative advantage curve is smaller, my model shows that productivity improvement in the tasks that a country is *not* performing actually has real benefits. It increases the incentives of the country to improve its productivity in tasks that it *is* currently performing.

Finally, since the terms of trade gains are more likely to happen in the offshoring context than in final goods trade context, my model thus points out that, as offshoring becomes more and more popular, countries are more likely to gain from other countries' productivity improvement.

Appendix

Appendix 1

Define $F(\bar{s}, e) \equiv \frac{t \int_0^{\bar{s}} c(s)ds}{et^* \int_{\bar{s}}^1 c^*(s)ds} - \frac{L}{L^*}$. When $F(\bar{s}, e) = 0$, it implicitly defines $\bar{s}(e)$. By implicit function theorem, we have,

$$\begin{aligned} \frac{d\bar{s}}{de} &= -\frac{\partial F/\partial e}{\partial F/\partial \bar{s}} \\ &= \left(e\left(\frac{c(\bar{s})}{\int_0^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^1 c^*(s)ds}\right) \right)^{-1} > 0. \end{aligned}$$

Notice that $\frac{d\bar{s}}{de}$ has nothing to do with $c^*(s)$ for $s < \bar{s}$.

Appendix 2

Substitute $\frac{d\bar{s}}{de} = \left(e\left(\frac{c(\bar{s})}{\int_{\bar{s}}^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^{1} c^*(s)ds}\right)\right)^{-1}$ into $\frac{d(w/w^*)}{de} = \frac{t^*}{t}\left(\frac{c^*(\bar{s})}{c(\bar{s})} + e\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\frac{d\bar{s}}{d\bar{s}}\right)$, we get $\frac{d(w/w^*)}{de} = \frac{t^*}{t}\left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}}{\frac{c}{\int_{\bar{s}}^{\bar{s}} c(s)ds} + \frac{\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}}{\int_{\bar{s}}^{\bar{s}} c(s)ds}\right).$

It is obvious that $\frac{d(w/w^*)}{de}$ may be positive, e.g. when $\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} = 0$. The object is to show that $\frac{d(w/w^*)}{de}$ might be negative for some functional forms of $\frac{c^*(\bar{s})}{c(\bar{s})}$.

Without loss of generality, we take a simple case where c(s) = 1 for $s \in [0, 1]$. In this case, $\frac{d\bar{s}}{de} = \left(e\left(\frac{1}{\bar{s}} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^1 c^*(s)ds}\right)\right)^{-1}$ and $\frac{d(w/w^*)}{de} < 0$ is equivalent to

$$-\frac{d(c^*(\bar{s}))}{d\bar{s}} > c^*(\bar{s}) \left(\frac{1}{\bar{s}} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^1 c^*(s)ds}\right).$$

Define x = 1 - s and $\tilde{c}^*(x) = c^*(1 - x)$, so $\tilde{c}^*(x)$ is an increasing function in $x \in [0, 1]$. Notice that $-\frac{d(c^*(s))}{ds} = -\frac{d(\tilde{c}^*(x))}{dx}\frac{dx}{ds} = \frac{d(\tilde{c}^*(x))}{dx}$, the above inequality can be rewritten as

$$\frac{d\left(\tilde{c}^{*}(\bar{x})\right)}{d\bar{x}} > \tilde{c}^{*}(\bar{x})\left(\frac{1}{1-\bar{x}} + \frac{\tilde{c}^{*}(\bar{x})}{\int_{0}^{\bar{x}}\tilde{c}^{*}(x)dx}\right).$$
(19)

For any given value of \bar{x} and any functional form $\tilde{c}^*(x)$ for $x \in [0, \bar{x}]$, The right hand side of the inequality is a finite positive number. We can always vary the functional form of $\tilde{c}^*(x)$ for $x \in (\bar{x}, 1]$ such that the inequality holds.

A more formal proof involves cubic bezier curves.¹⁵ Lemma 1 is equivalent to that for any given function $\hat{c}^*(x)$, $x \in [0,1]$, which is increasing, continuous, differentiable at any point $x \in [0,1]$, a function $\tilde{c}^*(x)$ can be constructed based on $\hat{c}^*(x)$ but differ from $\hat{c}^*(x)$ only locally around \bar{x} , such that the inequality (19) is satisfied at \bar{x} for $\tilde{c}^*(x)$.

¹⁵A cubic bezier curve is defined as $\mathbf{B}(\iota) = (1-\iota)\mathbf{B}_{\mathbf{P}_0,\mathbf{P}_1,\mathbf{P}_2}(\iota) + \tau \mathbf{B}_{\mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3}(\iota)$, $0 \le \iota \le 1$ where $\mathbf{B}_{\mathbf{P}_0,\mathbf{P}_1,\mathbf{P}_2}(\iota)$ and $\mathbf{B}_{\mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3}(\iota)$ are quadratic bezier curves, defined by points P_0 , P_1 , P_2 , and P_1 , P_2 , P_3 respectively and the points P_0 , P_1 , P_2 , and P_3 are points in a two-dimensional space. More specifically, a cubic bezier curve is $\mathbf{B}(\iota) = (1-\iota)^3\mathbf{P}_0 + 3(1-\iota)^2\iota\mathbf{P}_1 + 3(1-\iota)\iota^2\mathbf{P}_2 + \iota^3\mathbf{P}_3$, $0 \le \iota \le 1$, and a quadratic bezier curve is $\mathbf{B}_{\mathbf{P}_0,\mathbf{P}_1,\mathbf{P}_2}(\iota) = (1-\iota)[(1-\iota)\mathbf{P}_0 + \iota\mathbf{P}_1] + \iota[(1-\iota)\mathbf{P}_1 + \iota\mathbf{P}_2]$, $0 \le \iota \le 1$. The derivative respect to ι of a cubic bezier curve is: $\mathbf{B}'(\iota) = 3(1-\iota)^2(\mathbf{P}_1 - \mathbf{P}_0) + 6(1-\iota)\iota(\mathbf{P}_2 - \mathbf{P}_1) + 3\iota^2(\mathbf{P}_3 - \mathbf{P}_2)$, which is also a quadratic bezier curve.

Given any function $\hat{c}^*(x)$, $x \in [0, 1]$, that is increasing, continuous, differentiable. For any given point \bar{x} , define a small neighborhood, $U(\bar{x}, \varepsilon) \subset (0, 1)$. Two points are defined in this neighborhood, $P_0 = (\bar{x} - \varepsilon, \hat{c}^*(\bar{x} - \varepsilon))$ and $P_3 = (\bar{x}, \hat{c}^*(\bar{x}))$. As $\hat{c}^*(x)$ is increasing in x, we have $0 < \bar{x} - \varepsilon < \bar{x} < 1$ and $\hat{c}^*(0) < \hat{c}^*(\bar{x} - \varepsilon) < \hat{c}^*(\bar{x})$. We will construct a curve $\tilde{c}^*(x)$ such that $\tilde{c}^*(x) = \hat{c}^*(x)$ for $x \in [0, \bar{x} - \varepsilon]$, $\tilde{c}^*(\bar{x}) = \hat{c}^*(\bar{x})$ and satisfies the inequality (19) at \bar{x} .

In order for the inequality to hold at \bar{x} for $\tilde{c}^*(x)$, we notice that

$$RHS = \tilde{c}^*(\bar{x}) \left(\frac{1}{1 - \bar{x}} + \frac{\tilde{c}^*(\bar{x})}{\int_0^{\bar{x}} \tilde{c}^*(x) dx} \right) < \tilde{c}^*(\bar{x}) \left(\frac{1}{1 - \bar{x}} + \frac{\tilde{c}^*(\bar{x})}{\int_0^{\bar{x} - \varepsilon} \tilde{c}^*(x) dx} \right) = k$$

where k is constant as $\tilde{c}^*(x)$ for $x \in [0, \bar{x} - \varepsilon]$ is known.

Choosing any point P_1 and P_2 in the two dimensional space such that: 1) $P_2 = (x_2, y_2)$, in which $y_2 = k'(x_2 - \bar{x}) + c(\bar{x})$, k' > k and $x_2 < \bar{x}$, 2) $P_1 = (x_1, y_1)$ in which $x_1 > \bar{x} - \varepsilon$ and $y_1 = \tilde{c}_-^{*'}(\bar{x} - \varepsilon)(x_1 - (\bar{x} - \varepsilon)) + \tilde{c}^*(\bar{x} - \varepsilon)$, where $\tilde{c}_-^{*'}$ is the left derivative of \tilde{c}^* , and 3) $\mathbf{B}'(\iota) = 3(1-\iota)^2(\mathbf{P}_1 - \mathbf{P}_0) + 6(1-\iota)\iota(\mathbf{P}_2 - \mathbf{P}_1) + 3\iota^2(\mathbf{P}_3 - \mathbf{P}_2)$ is a vector with positive components, where $B(\iota)$ is defined as $\mathbf{B}(\iota) = (1-\iota)^3\mathbf{P}_0 + 3(1-\iota)^2\iota\mathbf{P}_1 + 3(1-\iota)\iota^2\mathbf{P}_2 + \iota^3\mathbf{P}_3$, $0 \le \iota \le 1$. Notice that $B'(\iota)$ has positive components is a sufficient but not necessary condition. A necessary condition is that point P_2 lies to the upper right of P_1 , which is easy to be satisfied.

Define function $\tilde{c}^*(x)$ in $x \in [\bar{x}-\varepsilon, \bar{x}]$ as the function defined by the "cubic bezier curve", B(t). By construction, $\tilde{c}_{-}^{*'}(\bar{x}) = k' > k$ (condition 1), $\tilde{c}_{-}^{*'}(\bar{x}-\varepsilon) = \tilde{c}_{+}^{*'}(\bar{x}-\varepsilon)$ (condition 2), and $\tilde{c}^*(x)$ is continuous and increasing in $x \in [\bar{x}-\varepsilon, \bar{x}]$ (condition 3). Thus, $\tilde{c}^*(x)$ is increasing, continuous, differentiable at any point $x \in [0, \bar{x}]$ and satisfy the inequality at \bar{x} .

By similar construction, we can have $\tilde{c}^*(x) = \hat{c}^*(x)$ for $x \in [\bar{x} + \varepsilon, 1]$, and construct $\tilde{c}^*(x)$ in the range of $x \in [\bar{x}, \bar{x} + \varepsilon]$ which satisfies $\tilde{c}^{*'}_+(\bar{x}) = k', \tilde{c}^{*'}_-(\bar{x} + \varepsilon) = \hat{c}^{*'}_+(\bar{x} + \varepsilon)$ and the derivative vector of the "cubic bezier curve" has positive components for $x \in [\bar{x}, \bar{x} + \varepsilon]$. In sum, $\tilde{c}^*(x)$ will be defined by $\tilde{c}^*(x) = \hat{c}^*(x)$ for $x \in [0, \bar{x} - \varepsilon]U[\bar{x} + \varepsilon, 1]$, and two "cubic bezier curves" for $x \in [\bar{x} - \varepsilon, \bar{x} + \varepsilon]$, and $\tilde{c}^*(x)$ satisfies the inequality (19) at point \bar{x} .

Appendix 3

The fact that home country welfare always increases as the technology transfer efficiency improves is obvious given that $\frac{d\omega}{d\bar{s}} = -\omega^2 t \int_{\bar{s}}^1 c^*(s) ds \frac{d\left(\frac{c(\bar{s})}{c^*(\bar{s})}\right)}{d\bar{s}} < 0$. For the foreign country, taking derivative of ω^* respect to e, we have

$$\begin{split} \frac{d\omega^*}{de} &= -\omega^{*2} \frac{d\left(e\left(\frac{t^*c^*(\bar{s})}{c(\bar{s})} \int_0^{\bar{s}} c(s)ds + t^* \int_{\bar{s}}^1 c^*(s)ds\right)\right)}{de} \\ &= -\omega^{*2} \left(\frac{t^*c^*(\bar{s})}{c(\bar{s})} \int_0^{\bar{s}} c(s)ds + t^* \int_{\bar{s}}^1 c^*(s)ds + e\frac{d\bar{s}}{de}\frac{t^*}{t}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \int_0^{\bar{s}} tc(s)ds\right) \\ &= -\omega^{*2} \int_0^{\bar{s}} tc(s)ds \left(\frac{t^*c^*(\bar{s})}{tc(\bar{s})} + \frac{\int_{\bar{s}}^1 t^*c^*(s)ds}{\int_0^{\bar{s}} tc(s)ds} + e\frac{d\bar{s}}{de}\frac{t^*}{t}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\right) \\ &= -\omega^{*2}t^* \int_0^{\bar{s}} c(s)ds \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{tL^*}{t^*eL} + e\frac{d\bar{s}}{de}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\right), \end{split}$$

where the last equality follows the trade balance equation, Equation (6).

The proof of Lemma 1 shows that there exist functional forms of $\frac{c^*(\bar{s})}{c(\bar{s})}$, such that $\frac{c^*(\bar{s})}{c(\bar{s})} + e\frac{d\bar{s}}{de}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} < 0$. A slight adjustment to the proof of Lemma 1, replacing $\frac{c^*(\bar{s})}{c(\bar{s})}$ by $\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{tL^*}{t^*eL}$ where $\frac{tL^*}{t^*eL}$ is a finite positive number, can show that $\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{tL^*}{t^*eL} + e\frac{d\bar{s}}{de}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} < 0$ for some functional forms of $\frac{c^*(\bar{s})}{c(\bar{s})}$. Combining this with Lemma 1, it is thus immediately followed that, we have finite numbers of n and m, which are defined as $0 < n \equiv \left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / \left(e\frac{d\bar{s}}{de}\right) < m \equiv \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{tL^*}{t^*eL}\right) / \left(e\frac{d\bar{s}}{de}\right)$. Explicitly, $n = \frac{c^*(\bar{s})}{c(\bar{s})} \left(\frac{c(\bar{s})}{\int_0^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^{1} c^*(s)ds}\right)$ and $m = \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{\int_{\bar{s}}^{1} c^*(s)ds}{\int_{0}^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^{1} c^*(s)ds}\right) \left(\frac{c(\bar{s})}{\int_{\bar{s}}^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^{\bar{s}} c^*(s)ds}\right)$. If $d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / d\bar{s}$ is in the range: 1) [-n, 0], then home relative wage increases, but foreign welfare still increases; 3) $[-\infty, -m]$, then home relative wage increases and foreign welfare still increases.

Appendix 4

Define $F(s, \tilde{t}^*) = \theta(s) - \frac{L}{L^*}$, where $\theta(s) = \frac{t \int_0^s c(j)dj}{\int_s^{\bar{s}} e\tilde{t}^* c^*(j)dj + \int_{\bar{s}}^1 e\tilde{t}^* c^*(j)dj}$. When $F(\bar{s}', \tilde{t}^*) = 0$, it implicitly defines $\bar{s}'(\tilde{t}^*)$. Note that

$$\frac{\partial F}{\partial \tilde{t^*}}\Big|_{\tilde{s}',\tilde{t^*}} = \frac{-et\int_0^{\tilde{s}'} c(j)dj\int_{\tilde{s}}^1 c^*(j)dj}{\left(\int_{\tilde{s}'}^{\tilde{s}} e\bar{t^*}c^*(j)dj + \int_{\tilde{s}}^1 e\tilde{t^*}c^*(j)dj\right)^2}$$

and

$$\frac{\partial F}{\partial \bar{s}'}\Big|_{\bar{s}',\tilde{t}^*} = \frac{tc(\bar{s}')\left(\int_{\bar{s}'}^{\bar{s}} e\bar{t}^*c^*(j)dj + \int_{\bar{s}}^{1} e\tilde{t}^*c^*(j)dj\right) + et\bar{t}^*c^*(\bar{s}')\int_{0}^{\bar{s}'} c(j)dj}{\left(\int_{\bar{s}'}^{\bar{s}} e\bar{t}^*c^*(j)dj + \int_{\bar{s}}^{1} e\tilde{t}^*c^*(j)dj\right)^2}.$$

By implicit function theorem, we have,

$$\begin{split} \frac{d\vec{s}}{d\tilde{t}^*} &= -\frac{\partial F/\partial t^*}{\partial F/\partial \vec{s}'} \\ &= \left(\frac{tc(\vec{s}')\left(\int_{\vec{s}'}^{\vec{s}} e\vec{t}^*c^*(j)dj + \int_{\vec{s}}^1 e\tilde{t}^*c^*(j)dj\right) + et\vec{t}^*c^*(\vec{s}')\int_{0}^{\vec{s}'}c(j)dj}{et\int_{0}^{\vec{s}'}c(j)dj\int_{\vec{s}}^1 c^*(j)dj}\right)^{-1} \\ &= \left(\frac{c(\vec{s}')}{\int_{0}^{\vec{s}'}c(j)dj}\left(\frac{\int_{\vec{s}'}^{\vec{s}} t^*c^*(j)dj}{\int_{\vec{s}}^1 c^*(j)dj} + \tilde{t}^*\right) + \frac{t^*c^*(\vec{s}')}{\int_{\vec{s}}^1 c^*(j)dj}\right)^{-1} > 0. \end{split}$$

Appendix 5

Home relative wage is increased when there is inframarginal technology improvement, as shown by Equation (12). To evaluate the impacts of inframarginal technology improvement on the welfare levels in the two countries, notice that the home welfare is given by

$$\omega = \left(\int_0^{\bar{s}'} tc(s)ds + \frac{w^*}{w} \left(\int_{\bar{s}'}^{\bar{s}} e\bar{t}^*c^*(s)ds + \int_{\bar{s}}^1 e\tilde{t}^*c^*(s)ds\right)\right)^{-1}$$

Taking derivative of ω respect to \tilde{t}^* , we have

$$\frac{d\omega}{d\tilde{t}^*} = -\omega^2 \left(\left(\int_{\bar{s}'}^{\bar{s}} e\bar{t}^* c^*(s) ds + \int_{\bar{s}}^1 e\tilde{t}^* c^*(s) ds \right) \frac{d\left(\frac{w^*}{w}\right)}{d\bar{s}'} \frac{d\bar{s}'}{d\tilde{t}^*} + \frac{ew^*}{w} \int_{\bar{s}}^1 c^*(s) ds \right) < 0.$$

For the foreign welfare, taking derivative of ω^* respect to \tilde{t}^* , we get

$$\frac{d\omega^*}{d\tilde{t}^*} = -\omega^{*2} \left(\frac{d\left(\frac{w}{w^*}\right)}{d\bar{s}'} \frac{d\bar{s}'}{d\tilde{t}^*} \int_0^{\bar{s}'} tc(s)ds + e \int_{\bar{s}}^1 c^*(s)ds \right).$$
(20)

The foreign inframarginal technology improvement again has two offsetting impacts on the foreign welfare. First, it unambiguously reduces the foreign relative wage and leads to welfare loss to the foreign country (the first term in the bracket). Second, it reduces the cost of performing tasks in the foreign and reduces the price of the final good. This contribute positively to the foreign welfare. The overall change of welfare in the foreign country depends on the functional form of $\frac{c^*(s)}{c(s)}$ at $s = \bar{s}'$.

From Equation (20), foreign welfare will increase if $\frac{d(\frac{w}{w^*})}{ds'} \frac{d\bar{s}'}{d\bar{t}^*} \int_0^{\bar{s}'} tc(s)ds + e \int_{\bar{s}}^1 c^*(s)ds > 0$, and otherwise decrease. Substitute in $\frac{d(\frac{w}{w^*})}{d\bar{s}'}$ and $\frac{d\bar{s}'}{d\bar{t}^*}$, the inequality is equivalent to

$$-\frac{d\left(\frac{c^{*}(\bar{s}')}{c(\bar{s}')}\right)}{d\bar{s}'} < \frac{\int_{\bar{s}}^{1} c^{*}(s)ds}{\bar{t}^{*}\int_{0}^{\bar{s}'} c(s)ds} \left(\frac{c(\bar{s}')}{\int_{0}^{\bar{s}'} c(s)ds} \left(\frac{\int_{\bar{s}'}^{\bar{s}'} \bar{t}^{*}c^{*}(s)ds}{\int_{\bar{s}}^{1} c^{*}(s)ds} + \tilde{t}^{*}\right) + \frac{\bar{t}^{*}c^{*}(\bar{s}')}{\int_{\bar{s}}^{1} c^{*}(s)ds}\right) \\ = \frac{c(\bar{s}')}{\left(\int_{0}^{\bar{s}'} c(s)ds\right)^{2}} \left(\int_{\bar{s}'}^{\bar{s}} c^{*}(s)ds + \frac{\tilde{t}^{*}}{\bar{t}^{*}}\int_{\bar{s}}^{1} c^{*}(s)ds\right) + \frac{c^{*}(\bar{s}')}{\int_{0}^{\bar{s}'} c(s)ds}.$$

Notice that the RHS of the inequality is not related to $c^*(s)$ for $s \in [0, \bar{s}')$, the inequality may or may not be satisfied, depending on the left derivative of the function $\frac{c^*(s)}{c(s)}$ at $s = \bar{s}'$. Defining $m' \equiv \frac{c(\bar{s}')}{\left(\int_{\bar{s}'}^{\bar{s}'} c(s)ds\right)^2} \left(\int_{\bar{s}'}^{\bar{s}} c^*(s)ds + \frac{\tilde{t}^*}{t^*}\int_{\bar{s}}^1 c^*(s)ds\right) + \frac{c^*(\bar{s}')}{\int_{\bar{s}'}^{\bar{s}'} c(s)ds} > 0$, we thus have

Defining $m \equiv \frac{c(s)}{\left(\int_{0}^{\overline{s}'} c(s)ds\right)^{2}} \left(\int_{\overline{s}'}^{\overline{s}'} c^{*}(s)ds + \frac{t}{t^{*}}\int_{\overline{s}}^{1} c^{*}(s)ds\right) + \frac{c(s)}{\int_{0}^{\overline{s}'} c(s)ds} > 0$, we thus have that, if $d\left(\frac{c^{*}(\overline{s}')}{c(\overline{s}')}\right)/d\overline{s}' \in [-m', 0]$, then foreign welfare will increase with inframarginal technology improvement and if $d\left(\frac{c^{*}(\overline{s}')}{c(\overline{s}')}\right)/d\overline{s}' \in [-\infty, -m']$, then foreign welfare will decrease.

Letting $\bar{s} = \bar{s}'$, $\bar{t}^* = \tilde{t}^* = t^*$, we have $m' = \frac{\int_{\bar{s}}^1 c^*(s)ds}{\int_0^{\bar{s}} c(s)ds} \left(\frac{c(\bar{s})}{\int_0^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^1 c^*(s)ds}\right)$. Notice that $m = \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{\int_{\bar{s}}^1 c^*(s)ds}{\int_0^{\bar{s}} c(s)ds}\right) \left(\frac{c(\bar{s})}{\int_0^{\bar{s}} c(s)ds} + \frac{c^*(\bar{s})}{\int_{\bar{s}}^1 c^*(s)ds}\right)$. We thus have m' < m when $\bar{s} = \bar{s}'$ and $\bar{t}^* = \tilde{t}^* = t^*$.

Appendix 6

Combining Equation (15) and Equation (2), we have

$$e = \frac{L^*}{L} \frac{t}{t^*} \left(\frac{c^*(\bar{s})}{c(\bar{s})} \right)^{-\sigma} \frac{\int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds}{\int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds}$$

As the right hand side of the equation is an increasing function in \bar{s} , for $\sigma \in (0, \infty)$, $\frac{d\bar{s}}{de} > 0$. Formally, taking derivative, we have

$$\frac{de}{d\bar{s}} = \frac{-e\sigma}{\frac{c^*(\bar{s})}{c(\bar{s})}} \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + e\left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_0^{\bar{s}}b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{1}b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right) > 0.$$

To see the impact on home relative wage, substituting $\frac{de}{d\bar{s}} = \left(\frac{d\bar{s}}{de}\right)^{-1}$ into Equation (7): $\frac{d(w/w^*)}{de} = \frac{t^*}{t} \left(\frac{c^*(\bar{s})}{c(\bar{s})} + e \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \frac{d\bar{s}}{de}\right), \text{ we obtain,}$

$$\begin{aligned} \frac{d(w/w^*)}{de} &= \frac{t^*}{t} \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}}{\frac{-\sigma}{c^*(\bar{s})} \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_0^{\bar{s}} b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{\bar{s}} b(s)^{\sigma}c^*(s)^{1-\sigma}ds} \right) \\ &= \frac{t^*}{t} \frac{c^*(\bar{s})}{c(\bar{s})} \left(\frac{\left(1 - \sigma\right) \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \frac{c^*(\bar{s})}{c(\bar{s})} \left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_0^{\bar{s}} b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{\bar{1}} b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)}{-\sigma \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \frac{c^*(\bar{s})}{c(\bar{s})} \left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_0^{\bar{s}} b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}ds}{\int_{\bar{s}}^{\bar{1}} b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)}\right) \end{aligned}$$

It is obvious that when $\sigma \in [1, \infty)$, $\frac{d(w/w^*)}{de} > 0$. In this case, the positive impact on home relative wage due to lower \bar{s} is always dominated by the negative impact due to direct reduction of e in the equilibrium relative wage. When σ goes to infinity, $\frac{d(w/w^*)}{de}$ goes to $\frac{t^*}{t} \frac{c^*(\bar{s})}{c(\bar{s})}$.

When $\sigma \in [0, 1)$, the direction of the home relative wage change is ambiguous and depends on the sign of $(1 - \sigma) \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \frac{c^*(\bar{s})}{c(\bar{s})} \left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_0^{\bar{s}}b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{\bar{s}}b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)$. A similar proof as the one for Lemma 1 (ignored here) can show that there exist functional forms of $\frac{c^*(\bar{s})}{c(\bar{s})}$ such that $(\sigma - 1) \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} > \frac{c^*(\bar{s})}{c(\bar{s})} \left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{\bar{s}}b(s)^{\sigma}c^*(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{\bar{s}}b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)$ and that $\frac{d(w/w^*)}{de} < 0$. Thus, the same statement as in Proposition 2 applies when $\sigma \in (0, 1)$.

Appendix 7

We have shown in the text that $d\omega/d\bar{s} < 0$ and $d\bar{s}/de > 0$. It follows that $d\omega/de < 0$. For the foreign real wage, rewriting the real wage as

$$\omega^* = \left((et^*)^{1-\sigma} \left(\left(\frac{c^*(\bar{s})}{c(\bar{s})} \right)^{1-\sigma} \int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds + \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds \right) \right)^{\frac{-1}{1-\sigma}}.$$

Taking derivative of ω^* respect to e, we have

$$\frac{d\omega^*}{de} = -\omega^{*2-\sigma} e^{-\sigma} t^{*1-\sigma} \int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds$$

$$\left(\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)^{1-\sigma} + \frac{\int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds}{\int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds} + e\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)^{-\sigma} \frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \frac{d\bar{s}}{de} \right)$$

Equation (2) and Equation (15) together give that

$$\frac{\int_{\bar{s}}^{1} b(s)^{\sigma} c^{*}(s)^{1-\sigma} ds}{\int_{0}^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds} = \frac{L^{*}}{L} \frac{t}{et^{*}} \left(\frac{c^{*}(\bar{s})}{c(\bar{s})}\right)^{-\sigma}$$

Substituting this into $d\omega^*/de$, we get

$$\frac{d\omega^*}{de} = -\omega^{*2-\sigma} t^{*1-\sigma} \int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds \left(\frac{ec^*(\bar{s})}{c(\bar{s})}\right)^{-\sigma} \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*} + e\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\frac{d\bar{s}}{de}\right)$$

Substituting in $e\frac{d\bar{s}}{de}$ as in Appendix 6, the terms in the bracket can be written as

$$\begin{split} & \frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*} + e^{\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\frac{d\bar{s}}{de}} \\ & = \frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*} + \left(\frac{-\sigma}{\frac{c^*(\bar{s})}{c(\bar{s})}}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_0^{\bar{s}}b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{1}b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)^{-1}\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} \\ & = \frac{\left((1-\sigma)\frac{c^*(\bar{s})}{c(\bar{s})} - \sigma\frac{L^*}{L}\frac{t}{et^*}\right)\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*}\right)\frac{c^*(\bar{s})}{c(\bar{s})}\left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{\int_{\bar{s}}^{\bar{s}}b(s)^{\sigma}c(s)^{1-\sigma}ds} + \frac{b(\bar{s})^{\sigma}c^*(\bar{s})^{1-\sigma}}{f^{\frac{1}{s}}b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)}{-\sigma\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}} + \frac{c^*(\bar{s})}{c(\bar{s})}\left(\frac{b(\bar{s})^{\sigma}c(\bar{s})^{1-\sigma}}{f^{\frac{1}{s}}b(s)^{\sigma}c^*(s)^{1-\sigma}ds}\right)} \end{split}$$

The sign of $\frac{d\omega^*}{de}$ thus depends on two factors. First, the magnitude of σ . It is easy to show that when $\beta \leq \sigma$, where $\beta = \left(\frac{c^*(\bar{s})}{c(\bar{s})}\right) / \left(\frac{c^*(\bar{s})}{c(\bar{s})} + \frac{L^*}{L}\frac{t}{et^*}\right) < 1$, then $(1 - \sigma)\frac{c^*(\bar{s})}{c(\bar{s})} - \sigma\frac{L^*}{L}\frac{t}{et^*} < 0$ and thus $\frac{d\omega^*}{de} < 0$ always. When the elasticity of substitution is greater than the threshold, β , foreign always gain from technology transfer improvement.

On the other hand, when $0 \leq \sigma < \beta$, the sign of $\frac{d\omega^*}{de}$ also depends on the slope of the comparative advantage curve. A similar proof as in the proof of Lemma 1 (ignored here) shows that if the slope of the curve is large enough, i.e. $\left|\frac{d\left(\frac{c^*(\bar{s})}{c(\bar{s})}\right)}{d\bar{s}}\right|$ is sufficiently big, then $\frac{d\omega^*}{de} > 0$ and the foreign country may be worse off from technology transfer efficiency improvement. When the slope of the curve is small, $\frac{d\omega^*}{de} < 0$ and the foreign country can still be better off.

Appendix 8

When $\sigma > 1$, Equation (17) is immediately followed by $d\left(\frac{w}{w^*}\right)/d\tilde{t}^* > 0$. Thus home relative wage decreases with lower \tilde{t}^* . To see the welfare changes, the home and foreign real wage can be written as, respectively,

$$\omega = \left(t^{1-\sigma} \int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds + \left(\frac{ew^* \tilde{t}^*}{w}\right)^{1-\sigma} \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds\right)^{\frac{-1}{1-\sigma}}$$

and

$$\omega^* = \left(\left(\left(\frac{tw}{w^*} \right)^{1-\sigma} \int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds + \left(e\tilde{t}^* \right)^{1-\sigma} \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds \right) \right)^{\frac{-1}{1-\sigma}}$$

By Equation (17), $\frac{w^*\tilde{t}^*}{w} = \tilde{t}^*\frac{1}{\sigma} \left(\frac{L^*}{L}\right)^{\frac{-1}{\sigma}} \left(\frac{t}{e}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\int_0^{\bar{s}} b(s)^{\sigma} c(s)^{1-\sigma} ds}{\int_{\bar{s}}^{1} b(s)^{\sigma} c^*(s)^{1-\sigma} ds}\right)^{\frac{-1}{\sigma}}$. Notice that the cutoff task \bar{s} will not be affected by reductions of \tilde{t}^* . It is thus obvious that $\frac{d\omega}{d\tilde{t}^*} < 0$ and that $\frac{d\omega^*}{d\tilde{t}^*} < 0$.

When $\sigma \in (0,1)$, from Equation (18) and (11), simple algebra reveals that that $\frac{d\bar{s}'}{d\bar{t}^*} > 0$. By Equation (11), the change of relative wage is given by Equation (12). Thus $d\left(\frac{w}{w^*}\right)/d\tilde{t}^* < 0$. I.e., inframarginal technology improvement always increases the home relative wage.

To evaluate the impacts of inframarginal technology improvement on the welfare levels, notice that the home real wage is given by

$$\begin{split} \omega^{\sigma-1} =& t^{1-\sigma} \int_0^{\bar{s}'} b(s)^{\sigma} c(s)^{1-\sigma} ds \\ &+ \left(\frac{ew^*}{w}\right)^{1-\sigma} \left(\tilde{t}^{*1-\sigma} \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds + \bar{t}^{*1-\sigma} \int_{\bar{s}'}^{\bar{s}} b(s)^{\sigma} c^*(s)^{1-\sigma} ds \right) \end{split}$$

and foreign real wage is

$$\begin{split} \omega^{*\sigma-1} &= \left(\frac{tw}{w^*}\right)^{1-\sigma} \int_0^{\bar{s}'} b(s)^{\sigma} c(s)^{1-\sigma} ds \\ &+ e^{1-\sigma} \left(\tilde{t}^{*1-\sigma} \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds + \bar{t}^{*1-\sigma} \int_{\bar{s}'}^{\bar{s}} b(s)^{\sigma} c^*(s)^{1-\sigma} ds\right). \end{split}$$

Taking derivative of ω respect to \tilde{t}^* , we have

$$\begin{aligned} \frac{d\omega}{d\tilde{t}^*} &= -\omega^{2-\sigma} e^{1-\sigma} \left(\int_{\bar{s}}^1 b(s)^{\sigma} \left(\tilde{t}^* c^*(s) \right)^{1-\sigma} ds + \int_{\bar{s}'}^{\bar{s}} b(s)^{\sigma} \left(\bar{t}^* c^*(s) \right)^{1-\sigma} ds \right) \left(\frac{w^*}{w} \right)^{-\sigma} \frac{d\left(\frac{w^*}{w} \right)}{d\tilde{t}^*} \\ &- \omega^{2-\sigma} e^{1-\sigma} \left(\frac{w^*}{w} \right)^{1-\sigma} \tilde{t}^{*-\sigma} \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds < 0. \end{aligned}$$

Similarly, taking derivative of ω^* respect to \tilde{t}^* , we get

$$\frac{d\omega^*}{d\tilde{t}^*} = -\omega^{*2-\sigma} \left(t^{1-\sigma} \left(\frac{w}{w^*} \right)^{-\sigma} \int_0^{\bar{s}'} b(s)^{\sigma} c(s)^{1-\sigma} ds \frac{d\left(\frac{w}{w^*} \right)}{d\tilde{t}^*} + e^{1-\sigma} \tilde{t}^{*-\sigma} \int_{\bar{s}}^1 b(s)^{\sigma} c^*(s)^{1-\sigma} ds \right)$$

The foreign inframarginal technology improvement has two offsetting impacts on the foreign welfare. First, it unambiguously increases the home relative wage and leads to welfare loss to the foreign country (the first term in the bracket). Second, it reduces the cost of performing tasks in the foreign and reduces the price of the final good. This contribute positively to the foreign welfare (the second term in the bracket).

Similar to the proof of Proposition 4, we can show that either one of the two effects might dominate, depending on the functional form of $\frac{c^*(s)}{c(s)}$ at $s = \bar{s}'$. If the slope of $\frac{c^*(s)}{c(s)}$ at $s = \bar{s}'$ is large, the foreign welfare will decrease and otherwise increase.

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