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ABSTRACT

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Which Model for Option Valuation in China? Empirical Evidence from SSE 50 ETF Options*

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Abstract

China launched its first exchange-traded option in early 2015 to meet the increasing demand for financial derivatives. In this paper, we provide an intensive empirical investigation among popular discrete time option pricing models in terms of their pricing performance when applied to SSE 50 ETF options. We find newly developed pricing models with realized measures significantly outperform the conventional GARCH-type models based on daily returns. In particular, our empirical results suggest that the leverage effect is very weak in the Chinese option market.

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1 Introduction

Options are one of the most important types of fundamental derivatives in global markets. They have been widely used in areas such as risk management and price discovery. Starting in the late 1990s, the Chinese stock market has experienced nearly 30 years of growth. The

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total market capitalization reached 56.62 trillion RMB by the end of 2017, making it the second largest stock market in the world. The inclusion of A shares in the MSCI index marks the growing importance of China's stock market to the global financial system and highlights the need for risk management instruments. On February 9, 2015, China launched its first exchange-based option-the Shanghai Stock Exchange (SSE) 50 ETF option. Unlike index options such as the S&P500 option, this option is written on SSE 50 ETF, which is the first and most liquid ETF in China's stock market. It covers 50 blue chip stocks listed on the SSE. The trading volume and open interest have increased nearly 70 fold over the past three years and the number of qualified investors has increased 100 fold[1]. This fast-growing market needs reliable pricing models.

Black and Scholes (1973) proposed the Black-Scholes formula for option pricing based on the constant volatility geometric Brownian motions. However, studies since the 1978 market crash have rejected the constant volatility assumption. Newer models highlight the modeling of dynamic volatility as the core of option pricing. Other than continuous time models such as the one proposed by Heston (1993), researchers motivated by the success of GARCH models in financial econometrics have investigated ways that GARCH models can be applied to option valuation. Duan (1995) pioneered this area by introducing a possible transition between physical and risk-neutral dynamics for GARCH models [2]. Among early studies of GARCH option pricing, Heston and Nandi (2000) made a significant contribution by proposing a GARCH structure with a close form pricing formula for European options. Christoffersen and Jacobs (2004) discussed a list of GARCH models in terms of their pricing performance on S&P 500 index options. Other specifications in GARCH models such as volatility components (Christoffersen et al. (2008)), non-normality (Christoffersen et al. (2010)), etc. have also been discussed. Discrete time models are easier to estimate than continuous time models and are time efficient for pricing large-scale option panels. In addition to LRNVR, risk neutralization methods such as the variance dependent pricing kernel method (Christoffersen et al. (2013)) are also used in GARCH option pricing.

Over the past two decades, high-frequency financial data have drawn increasing attention from researchers. A variety of realized measures such as the realized variance (Ander-

[1] The number of qualified investors on the first trading day in 2015 was 2,626. There were 26,0040 by early 2018.

[2] The transformation is called the locally risk-neutral valuation relationship (LRNVR).

sen and Bollerslev (1998)), realized bipower-variation (Barndorff-Nielsen (2004)), realized kernel (Barndorff-Nielsen et al. (2008)), etc. have been used to provide model-free estimations of daily variance. Based on these model-free measures, reduced form models such as the ARFIMA model (Andersen et al. (2003)) and complete models such as the Realized GARCH model (Hansen et al. (2012), Hansen and Huang (2016)) have been proposed to model volatility dynamics. With the help of realized measures, these models can identify the volatility-specific shocks and assign them risk premium parameters based on the exponential affine stochastic pricing factor. Major GARCH-type models tailored for option pricing include the GARV model (Christoffersen et al. (2014)), the Realized GARCH model (Huang et al. (2017)), etc. The types of model used have been extended beyond GARCH by Majewski et al. (2015), who introduced a LHARG model based on the popular reduced-form HAR model (Corsi (2009)). All of these studies highlighted the importance of realized measures in improving the pricing performance of discrete time models.

Although previous studies have made important contributions, several studies have focused on comparing different option pricing models to help guide model design. For example, Christoffersen and Jacobs (2004) compared the option pricing performance of several common GARCH models and highlighted the importance of the leverage effect and joint estimation for both underlying and option data. However, most studies that have compared option pricing models (especially discrete time models) focus on U.S. dataset, and little attention has been paid to this issue in a Chinese context. The Chinese market is different from the U.S. market in many ways. The option is a physically delivered ETF option rather than a cash settled index option. In the Chinese market, the strikes are quite limited, the time to maturity is shorter, and the cost of short selling is much higher than in the U.S. market. Therefore, an extensive comparison of (discrete time) option pricing models, including models with high-frequency data-based volatility measures, provides both academics and practitioners valuable guidelines for choosing option pricing models in China.

Therefore, we intensively test a set of common discrete time volatility models that have been used for option pricing in previous studies. The models include standard GARCH models and realized measure based models in which risk neutralization is done with LRNVR, variance dependent pricing kernel, and the exponential affine stochastic discount factor. For those models without explicit option prices, Duan et al. (1999)'s method is used to determine an analytical approximated price. This technique has been tailored to the Realized GARCH model used by Huang et al. (2017). All of the parameters are estimated with option-underlying joint estimation and both in-sample and out-of-sample results are discussed.

The results show that discrete time models can be applied to price SSE 50 ETF options with reasonable pricing errors. In line with previous studies, we find the realized measure-based models have better in-sample and out-of-sample pricing performance over every kind of option. The complex realized measures, although they are preferred in modeling and forecasting volatility, do not provide better pricing results than simple realized measures like the conventional realized variance. However, the Chinese data provide us with some surprising results. The first is that although the leverage effect has been shown to have a strong effect on the pricing of U.S. index options, it has much less effect on the modeling of Chinese data. Second, the first year of trading in the Chinese market (2015), which suffers from both extreme market volatility and severely limited arbitrage, is distinct from subsequent years, suggesting that the first year needs to be either dropped from the sample or treated separately.

The remainder of this paper is organized as follows. In Section 2, we provide the list of models used in the comparison. In Section 3, we briefly introduce the estimation method used. In Section 4, we present and discuss our empirical results. The last section concludes the paper.

2 Model in comparison

In this study, we test nine discrete time volatility models including four GARCH models (a standard GARCH model (GARCH, Engle and Bollerslev (1986)) and three aysmmetric GARCH models (GJR-GARCH, Glosten et al. (1993); NGARCH, Engle and Ng (1993); EGARCH, Nelson (1991))); two Heston-Nandi GARCH models (HNG, Heston and Nandi (2000); HNGvd, Christoffersen et al. (2013))[3]; and three models with realized measures (Realized GARCH (Hansen and Huang (2016)), GARV (Christoffersen et al. (2014)), and LHARG (Majewski et al. (2015))). Table 1 describes the differences between these models.

[3]HNGvd in this study refers to a model with a variance-dependent pricing kernel (Christoffersen et al. (2013)). The model is based on the original Heston-Nandi GARCH model, but the risk neutralization method differs from the conventional LRNVR method proposed by Duan (1995).

[Insert Table 1 here]

GARCH models

The GARCH models selected in this study are based on the following GARCH-in-mean framework:

$$r_{t+1} \quad = \quad r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}$$

where z_t follows a standard normal distribution. The parameter λ measures the required return of an investor that is proportional to the conditional volatility. The variance equations for the different models are as follows:

- Standard GARCH : $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t z_t^2$
- GJR-GARCH : $h_{t+1} = \beta_0 + h_t\left[\beta_1 + \tau_1 z_t^2 + \tau_2 I_{\{z_t<0\}} z_t^2\right]$
- NGARCH : $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t\left(z_t - \tau_2\right)^2$
- EGARCH : $\log h_{t+1} = \beta_0 + \beta_1 \log h_t + \tau_1 z_t + \tau_2\left(|z_t| - \sqrt{\frac{2}{\pi}}\right)$

The first three models are linear models, whereas the fourth is a log-linear model. Unlike the linear GARCH models, the log-linear GARCH model uses the standardized shock z_t instead of the nonstandard shock $\sqrt{h_t}z_t$ to drive the volatility process. It also imposes fewer constraints on the parameters to guarantee a positive conditional variance. These advantages come at the cost of a tendency to overreact to volatility shocks and a much more complicated multi-period volatility forecast formula.

Following Duan (1995) and others, the risk neutral dynamics are linked to their physical counterparts with a locally risk-neutral valuation relationship. Thus, the corresponding risk neutral dynamics are

$$r_{t+1} \quad = \quad r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^*$$

where z_t^* follows a standard normal distribution. The variance equations for the different models are as follows.

- Standard GARCH: $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t\left(z_t^* - \lambda\right)^2$

- GJR-GARCH: $h_{t+1} = \beta_0 + h_t \left[\beta_1 + \tau_1 (z_t^* - \lambda)^2 + \tau_2 I_{\{z_t^* < \lambda\}} (z_t^* - \lambda)^2\right]$
- NGARCH: $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t \left(z_t^* - \lambda - \tau_2\right)^2$
- EGARCH: $\log h_{t+1} = \beta_0 + \beta_1 \log h_t + \tau_1 \left(z_t^* - \lambda\right) + \tau_2 \left(|z_t^* - \lambda| - \sqrt{\frac{2}{\pi}}\right)$

As all of these models are non-affine models[4], and the traditional close form pricing formula via Fourier inverse transformation is not available. Here, we follow Duan et al. (1999) and price the European call options using an analytical approximation[5]. It is worth noting that the EGARCH models are much slower than the linear GARCH models when the terms needed for the approximation are calculated.

Heston-Nandi GARCH models

Unlike the GARCH models discussed in the previous section, the Heston-Nandi GARCH (Heston and Nandi (2000)) is an affine model with an explicit moment generation function that can be used to calculate close-form option prices. This feature makes it a popular benchmark model in discrete time option pricing. The mean equation is

$$r_{t+1} = r + \left(\lambda - \frac{1}{2}\right) h_{t+1} + \sqrt{h_{t+1}} z_{t+1}$$

where z_t follows standard normal distribution. The parameter λ measures the required return of an investor that is proportional to the conditional volatility. The variance equation is specified as

$$h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 \left(z_t - \tau_2 \sqrt{h_t}\right)^2$$

The risk neutralization of this model can be done in two different ways. The first method is Duan's LRNVR (In this paper, we refer to the Heston-Nandi GARCH with LRNVR as HNG.). The second method relies on the variance-dependent pricing kernel proposed by Christoffersen et al. (2013). The latter method explicitly provides an additional parameter

[4]In an affine model the moment generation function $E_0(e^{\phi X_t})$ of the return process $X_t = \log(S_t/S_0)$ is an exponential linear function.

[5]The GARCH model can be viewed as a special case ($\tau_2 = 0$) of NGARCH/CJR-GARCH. The approximation formula can be adapted from the one for NGARCH/GJR-GARCH by using the constraint $\tau_2 = 0$.

in the pricing kernel to accommodate variance risk premium (In this paper, we refer to the Heston-Nandi GARCH with a variance-dependent pricing kernel as HNGvd.). The corresponding risk neutral dynamics are

$$
\begin{aligned}
\text{HNG} : r_{t+1} &= r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z^*_{t+1} \\
\text{HNGvd} : r_{t+1} &= r - \frac{1}{2}h^*_{t+1} + \sqrt{h^*_{t+1}}z^*_{t+1}
\end{aligned}
$$

where z^*_t follows a standard normal distribution. The variance equations are as follows:

- HNG: $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 \left(z^*_t - (\tau_2 + \lambda)\sqrt{h_t}\right)^2$
- HNGvd: $h^*_{t+1} = \beta^*_0 + \beta_1 h^*_t + \tau^*_1 \left(z^*_t - \tau^*_2 \sqrt{h^*_t}\right)^2$

where $h^*_t = \tilde{\xi} h_t$, $\beta^*_0 = \tilde{\xi}\beta_0$, $\tau^*_1 = \tilde{\xi}^2\tau_1$, $\tau^*_2 = 1/2 + (\tau_2 + \lambda - 1/2)/\tilde{\xi}$ and $\tilde{\xi} = 1/(1 + 2\xi\tau_1)$. Parameter ξ is the free parameter in the variance-dependent pricing kernel.

As the model structures of HNGvd and HNG are the same under risk neutral dynamics, the moment generation function provided in Heston and Nandi (2000) can be adapted for both models. The European option prices can be calculated using the Fourier inverse transformation.

Models with realized variance

Several models have been proposed for high frequency data-based volatility modeling. Within the GARCH framework, the Realized GARCH (Hansen et al. (2012), Hansen and Huang (2016)), MEM (Engle and Gallo (2006)), and HEAVY (Shephard and Sheppard (2010)) are commonly used complete models that can jointly model returns and realized variance. Reduced models such as HAR (Corsi (2009)) are receiving increasing attention. In this study, we focus on three models that have been adapted to option pricing practice.

- **Realized GARCH**

The Realized GARCH model was proposed by Hansen et al. (2012) as an extension of GARCH-X. Hansen and Huang (2016) introduced the realized exponential GARCH, which describes returns and realized variance joint dynamics as follows:

$$\begin{aligned}
r_{t+1} &= r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1} \\
\log h_{t+1} &= \omega + \beta \log h_t + \tau_1 z_t + \tau_2(z_t^2 - 1) + \gamma\sigma u_t \\
\log x_t &= \xi + \phi \log h_t + d_1 z_t + d_2(z_t^2 - 1) + \sigma u_t
\end{aligned}$$

where z_t and u_t are independent standard normal random variables. The volatility-specific shock u_t enables the model to accommodate variance risk premium in addition to equity premium. The last "measurement equation" links u_t with the realized variance and makes the simple ML estimator available.

Following Christoffersen et al. (2010) and others, we use the exponential affine stochastic discount factor to transform the model into its risk neutral counterpart:

$$\begin{aligned}
r_{t+1} &= r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z^*_{t+1} \\
\log h_{t+1} &= \omega^* + \beta \log h_t + \tau_1^* z_t^{*2} + \tau_2(z_t^* - 1) + \gamma\sigma u_t^* \\
\log x_t &= \xi^* + \phi \log h_t + d_1^* z_t^* + d_2(z_t^{*2} - 1) + \sigma u_t^*
\end{aligned}$$

where z_t^* and u_t^* are independent standard normal random variables. $\omega^* = \omega + \gamma\sigma\chi + \lambda\tau_2(\lambda - 1)$, $\xi^* = \xi + \sigma\chi + \lambda d_2(\lambda - 1)$, $\tau_1^* = \tau_1 - 2\lambda\tau_2$, and $d_1^* = d_1 - 2\lambda d_2$. χ are the parameters associated with the variance risk premium, which is introduced into the model via the discount factor. As the Realized GARCH model is not an affine model, the conventional close-form pricing formula is not available. Huang et al. (2017) provided an alternative pricing method with an analytical approximation. The idea was to expand the distribution of the cumulative return with normal distributions and analytical higher moments. See Proposition 1 in their paper for the detailed pricing formula.

- **GARV**

Another complete model dedicated to price options under the GARCH framework is the Generalized Affine Realized Volatility (GARV) model proposed by Christoffersen et al. (2014). Unlike the Realized GARCH model, the GARV model decomposes the variance into two parts: the variance calculated via the daily return h^R_{t+1} and the variance calculated through the realized variance h_{t+1}^{RV}. Each of the individual series follows a Heston-Nandi type of innovation and a measurement equation is added to close the model:

$$\begin{aligned}
r_{t+1} &= r + \left(\lambda - \frac{1}{2}\right)\bar{h}_{t+1} + \sqrt{\bar{h}_{t+1}}z_{t+1} \\
\bar{h}_{t+1} &= \kappa h_{t+1}^{R} + (1-\kappa)h_{t+1}^{RV} \\
h_{t+1}^{R} &= \omega + \beta h_t^{R} + \tau_1\left(z_t - \tau_2\sqrt{\bar{h}_t}\right)^2 \\
h_{t+1}^{RV} &= \xi + \phi h_t^{RV} + d_1\left(\epsilon_t - d_2\sqrt{\bar{h}_t}\right)^2 \\
RV_t &= h_t^{RV} + \alpha_2\left[\epsilon_t^2 - 1 - 2d_2\epsilon_t\sqrt{\bar{h}_t}\right]
\end{aligned}$$

where (z_t, ϵ_t) follows a standard bi-variate normal distribution with correlation ρ. We also introduce $\gamma = d_1/\alpha_2$. The risk neutral dynamics under the exponential affine stochastic discount factor are

$$\begin{aligned}
r_{t+1} &= r - \frac{1}{2}\bar{h}_{t+1} + \sqrt{\bar{h}_{t+1}}z_{t+1}^{*} \\
\bar{h}_{t+1} &= \kappa h_{t+1}^{R} + (1-\kappa)h_{t+1}^{RV} \\
h_{t+1}^{R} &= \omega_1 + \beta_1 h_t^{R} + \tau_1\left(z_t - \tau_2^{*}\sqrt{\bar{h}_t}\right)^2 \\
h_{t+1}^{RV} &= \xi + \phi h_t^{RV} + d_1\left(\epsilon_t^{*} - d_2^{*}\sqrt{\bar{h}_t}\right)^2 \\
RV_t &= h_t^{RV} + \alpha_2\left[\epsilon_t^{*2} - 1 - 2d_2^{*}\epsilon_t^{*}\sqrt{\bar{h}_t}\right]
\end{aligned}$$

where $(z_t^{*}, \epsilon_t^{*})$ follows a standard bi-variate normal distribution with correlation ρ. $\tau_2^{*} = \tau_2 + \lambda$, and $d_2^{*} = d_2 - \chi$. χ is the parameter associated with the variance risk premium that is introduced into the model via the discount factor. Due to the affine structure of the GARV model, a close-form solution is available and provided in Christoffersen et al. (2014).

- **LHARG**

In addition to the GARCH-type models, the availability of high-frequency data and realized measures have boosted the development of reduced form models such as the HAR model (Corsi (2009)). In particular, the HAR model is adapted using Heston-Nandi type leverage functions and a gamma distribution (LHARG) to price European call options. Majewski et al. (2015) provided a general framework of option pricing with a LHARG model. In this study, we use an extended LHARG by adding quarterly and yearly data to better model the long-memory feature of volatility. A similar extension is used in Huang et al. (2018) for VIX futures pricing to improve LHARG's performance in pricing long maturity futures. The dynamics under physical measures are

$$R_{t+1} = r + \lambda RV_{t+1} - \frac{1}{2}RV_{t+1} + \sqrt{RV_{t+1}}\epsilon_{t+1}, \quad \epsilon_{t+1} \sim i.i.d\ N(0,1)$$

$$RV_{t+1}|\mathcal{F}_t \sim \Gamma(\delta, \Theta(\boldsymbol{RV_t}, \boldsymbol{L_t}), \theta)$$

and

$$\Theta(\boldsymbol{RV_t}, \ell_t) = d + \beta_d RV_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_q RV_t^{(q)} + \beta_y RV_t^{(y)} + \alpha_d \ell_t$$

We define the components as follows:

$$RV_t^{(d)} = RV_t, \quad RV_t^{(w)} = \tfrac{1}{4}\textstyle\sum_{i=1}^{4} RV_{t-i}, \quad RV_t^{(m)} = \tfrac{1}{17}\textstyle\sum_{i=5}^{21} RV_{t-i},$$

$$RV_t^{(q)} = \tfrac{1}{41}\textstyle\sum_{i=22}^{62} RV_{t-i}, \quad RV_t^{(y)} = \tfrac{1}{189}\textstyle\sum_{i=63}^{251} RV_{t-i},$$

and

$$\ell_t = \epsilon_t^2 - 1 - 2\gamma\epsilon_t\sqrt{RV_t}$$

ℓ_t is the leverage term that describes the asymmetric reaction of volatility in response to positive and negative shocks from returns. Following Huang et al. (2018), we add daily leverage to keep the model more concise. Adding additional components does not change the main results. The risk neutral dynamics under the exponential affine stochastic discount factor is

$$R_{t+1} = r - \frac{1}{2}RV_{t+1} + \sqrt{RV_{t+1}}\epsilon^*_{t+1}, \quad \epsilon^*_{t+1} \sim i.i.d\ N(0,1)$$

$$RV_{t+1}|\mathcal{F}_t \sim \Gamma(\delta, \Theta^*(\boldsymbol{RV_t}, \boldsymbol{L^*_t}), \theta^*)$$

and

$$\Theta^*(\boldsymbol{RV_t}, \ell^*_t) = d^* + \beta^*_d RV_t^{(d)} + \beta^*_w RV_t^{(w)} + \beta^*_m RV_t^{(m)} + \beta^*_q RV_t^{(q)} + \beta^*_y RV_t^{(y)} + \alpha^*_d \ell^*_t$$

The stared risk-neutral parameters are linked to the physical parameters as follows:

$$\beta_d^* = \Delta[\beta_d + \alpha_d(2\gamma\lambda + \lambda^2)], \qquad \beta_j^* = \Delta\beta_j \;\text{ for } j \in \{w, m, q, y\},$$

$$\alpha_d^* = \Delta\alpha_d, \qquad d^* = \Delta d, \qquad \theta^* = \Delta\theta, \qquad \gamma^* = \gamma + \lambda,$$

and

$$\ell_t^{(d)} = \epsilon_t^{*2} - 1 - 2\gamma^*\epsilon_t^*\sqrt{RV_t}$$

where $\Delta = \{1 + \theta[(\lambda - 1/2)^2/2 - \chi - 1/8]\}^{-1/2}$. Again, χ is the parameter associated with the variance risk premium. Majewski et al. (2015) provided the option pricing formula for arbitrary lags, which can be easily adapted to our setting.

3 Estimation Method

In previous studies of option pricing, the parameters have been estimated using several different methods. The most direct way to calibrate the parameters (often referred to as the nonlinear least square (NLS) method) is to minimize the mean square error between the model price and the corresponding market price or the Vega weighted pricing error[6]. However, NLS tends to distort the model parameters to extreme values. To overcome this drawback, recent studies have turned to the joint estimation method, which takes model's fit of physical dynamics into account. As the discrete time models under physical measures can be estimated using the maximum likelihood method, the joint estimation under discrete time models has been receiving increasing attention from researches (e.g., Christoffersen et al. (2014), Huang et al. (2017)). In this study, we estimate the parameters using joint estimations with corresponding log-likelihood functions.

Log-likelihood for underlying process

The log-likelihood for the underlying process measures a model's ability to describe the physical dynamics of the returns and realized measures (if applicable).

[6]According to the definition of Vega, this pricing error is an approximation of the error in the implied volatility.

- Leveraged GARCH models

$$\ell_R = -\frac{T}{2}\log(2\pi h_t) - \frac{1}{2}\left\{\sum_{t=1}^{T}(r_t - r - \lambda\sqrt{h_t} - \frac{1}{2}h_t)^2/h_t\right\}$$

- Linear affine GARCH models

$$\ell_R = -\frac{T}{2}\log(2\pi h_t) - \frac{1}{2}\left\{\sum_{t=1}^{T}(r_t - r - (\lambda - \frac{1}{2})h_t)^2/h_t\right\}$$

- Realized GARCH

$$\ell_R = -\frac{T}{2}\log(2\pi h_t) - \frac{1}{2}\left\{\sum_{t=1}^{T}(r_t - r - (\lambda - \frac{1}{2})h_t)^2/h_t\right\}$$
$$\ell_{RV} = -\frac{T}{2}\log(2\pi\sigma^2) - \frac{1}{2}\left\{\sum_{t=1}^{T}(\log x_t - \xi - \phi\log h_t - d_1 z_t - d_2(z_t^2 - 1))/\sigma^2\right\}$$

- GARV

  As the exact likelihood function for the volatility shock in GARV is hard to obtain, the QMLE method is applied by assuming a bivariate normal distribution of (z_t, ϵ_t):

$$\mu_{t+1} = \begin{bmatrix} \mu_{t+1}^{R} \\ \mu_{t+1}^{RV} \end{bmatrix} = \begin{bmatrix} r + (\lambda - \frac{1}{2})\bar{h}_{t+1} \\ h_{t+1}^{RV} \end{bmatrix}, \quad \mathbf{\Sigma}_{t+1} = \begin{bmatrix} \bar{h}_{t+1} & -2\rho d_2\alpha_2\bar{h}_{t+1} \\ -2\rho d_2\alpha_2\bar{h}_{t+1} & 2\alpha_2^2(1 + 2d_2^2\bar{h}_{t+1}) \end{bmatrix}$$

  and

$$\ell_{R,RV} = -T\log(2\pi) - \frac{T}{2}\log(|\Sigma|) - \sum_{t=2}^{T}\frac{(\mathbf{x}_t - \mu_t)^T\Sigma_t^{-1}(\mathbf{x} - \mu_t)}{2}$$

  where $\mathbf{x}_t = (r_t, x_t)^T$.

- LHARG

$$\ell_R = -\frac{T}{2}\log(2\pi RV_t) - \frac{1}{2}\left\{\sum_{t=1}^{T}(r_t - r - (\lambda - \frac{1}{2})RV_t)^2/RV_t\right\}$$
$$\ell_{RV} = -\sum_{t=1}^{T}\Big(\frac{RV_t}{\theta} + \Theta(\boldsymbol{RV_{t-1}}, \boldsymbol{L_{t-1}})\Big) + \sum_{t=1}^{T}\log\Big(\sum_{k=0}^{\infty}\frac{RV_t^{\delta+k-1}}{\theta^{\delta+k}\Gamma(\delta+k)}\frac{\Theta(\boldsymbol{RV_{t-1}}, \boldsymbol{L_{t-1}})^k}{k!}\Big)$$

Log-likelihood for pricing error

In this study, the option pricing error is defined as the Vega weighted pricing error to mimic the difference in the implied volatility:

$$e_i = \frac{O_i^{Mod} - O_i^{Mkt}}{Vega_i}$$

where O_i^{Mod} and O_i^{Mkt} are the model and market option price for option i, respectively. The option pricing formula for the European options can be found in the following papers: Duan et al. (2006) for GARCH, EGARCH, NGARCH, and CJR-GARCH; Heston and Nandi (2000) for HNG; Christoffersen et al. (2013) for HNGvd; Huang et al. (2017) for RG; Christoffersen et al. (2014) for GARV, and Majewski et al. (2015) for LHARG.

We assume that the weighted pricing error follows the normal distribution $N(0, \sigma_e^2)$, and the corresponding log-likelihood function is

$$\ell_o = -\frac{N}{2}\log(2\pi\sigma_e^2) - \frac{1}{2}\left\{\sum_{t=1}^{N}(e_i)^2/\sigma_e^2\right\}$$

.

Joint log-likelihood

The joint log-likelihood is constructed by adding the log-likelihood of the underlying and pricing errors together:

$$\ell = \ell_{R,RV} + \ell_o$$

where $\ell_{R,RV} = \ell_R + \ell_{RV}$ for RG and LHARG and set $\ell_{RV} = 0$ for models without realized measures.

4 Empirical Results

4.1 The Dataset

The SSE 50 ETF option was launched on February 9, 2015 and experienced rapid growth in the subsequent years. The daily trading volume and open interest increased nearly 70 fold drom 2015 to 2018. Figure 1 provides an overview of the market. As a new derivative in an emerging market, it has several notable features.

[Insert figure 1 here]

During the first year of trading, the market experienced the bust of stock bubbles in mid-2015, the restart of IPO in late-2015 and the week of shutdown mechanism in early-2016. All of them created significant fluctuation in the market and we conduct our empirical investigation with both full sample and post 2015 sub-sample.

The second feature is the limited number of contracts. The SSE 50 ETF options initially had five strikes (1 ATM, 2 OTM, and 2 ITM), which had increased to nine strikes (1 ATM, 4 OTM, 4 ITM) by the beginning of 2018. This limitation makes the conventional option pricing using only Wednesday data impractical for studies using China's dataset. Therefore, we use option prices from all of the available trading days.

The third feature is that the short sell cost implied by option prices are high in 2015 and decrease over time. Put-call parity is one of the simplest and best known no-arbitrage relations. No deviations from no-arbitrage means that the put-call parity implied dividend yield should be close to the actual dividend yield, and a relatively higher implied dividend yield signifies short sale constraints on the underlying assets[7]. Following Ofek et al. (2004) and Bilson et al. (2015), we derive the put-call parity implied dividend yield and compare it with the actual dividend yield. Specifically, for each option pairs i with the same strike and maturity on day t, we derive an implied dividend yield $y_i(t)$ (annualized) using the put-call parity:

$$C_i(t) - P_i(t) = S(t)e^{-y_i(t)T} - Ke^{-rT}$$

where $C_i(t)$ and $P_i(t)$ are the prices of a call and a put option price pairs with the same maturity T and the same strike price K. r is the risk-free rate. There are N_t option pairs on day t with the same strike and maturity. Figure 1(d) reports the daily average implied dividend yield $y(t) = \frac{1}{N_t}\sum_{i=1}^{N_t} y_i(t)$ and the historical average dividend yield. We can find that implied dividend yields are higher than the actual value in most cases, especially in the second half of 2015 due to the stock market crash that year. This finding is related to the high short sell cost in China. The gap between the implied and actual dividend yield decreases over time, indicating that the option market has become more efficient in recent

[7] The put-call parity implied dividend yield is a proxy for the actual borrow rate of the underlying asset.

years. This finding suggests that conventional option pricing methods that only use call (or put) option data give biased results. Therefore, we use both call and put option data.

Accordingly, we trim our raw dataset covering the 2015/02-2018/02 period with the following procedures.

1. Options that do not satisfy the arbitrage restriction are dropped. As an option price is adjusted when the dividend is paid (dividend protected), the SSE 50 ETF options can be treated as European options without dividends [8]. The arbitrage restrictions are set as

$$C(t) \geq \max(0, S(t) - Ke^{-rT})$$
$$P(t) \geq \max(0, Ke^{-rT} - S(t))$$

2. Options with zero trading volume are dropped.

3. Options with maturities shorter than 5 or longer than 90 are dropped. In our dataset, options with maturities less than 90 days account for 92% of the total trading volume.

4. Options with low liquidity are dropped. That means, for every maturity on a given day, we drop the options with trading volumes lower than the median volume of the group.

The resulting dataset contains 12,281 option prices. Table 2 provides an overview of the dataset with the number of prices and the average implied volatility. From Panel A, we find that, unlike the U.S. market, the Chinese market has a relatively balanced volatility smile rather than a volatility smirk. The balanced feature indicates relatively weaker effect of the leverage effect on the model. Panel B shows a flat or even downward sloping volatility term structure. Also, most of the maturities are 60 days or less, indicating a lower demand for long-memory structures in the pricing model.

[Insert Table 2 here]

[8] For more information, please refer to http://english.sse.com.cn/products/derivative

4.2 Estimated Parameters

Table 3 provides the parameter estimations for different models using the full sample from February 2015 to February 2018. The first eight models are GARCH type models and share the same notations, if applicable. The last LHARG model has a different set of parameters and we indicate them accordingly.

[Insert Table 3 here]

Several commonly mentioned features can be found. 1) All of the models have a highly persistent volatility process under both physical and risk neutral measures. 2) Most of the models have a positive and significant equity premium parameter (λ). The models with explicit volatility risk premium parameters ($\chi/\tilde{\xi}$) indicate a higher risk neutral volatility than their physical counterparts.

However, we also find an unconventional positive leverage effect (i.e., given the same magnitude of shock, a positive shock induces higher volatility in the next period) for most models, probably because of the leverage boom in early 2015 when return and volatility were highly positively correlated. If we estimate parameters without the data from 2015, only weak correlations are found in the GARCH models. Generally, the conventional leverage effect is weak in the Chinese market.

4.3 In-sample Pricing Performance

Table 4 provides the in-sample pricing performance across different models. The performance is evaluated by the mean square error of implied volatility (IVRMSE):

$$IVRMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[IV_i^{Mod} - IV_i^{Mkt}\right]^2} \times 100$$

where IV_i^{Mod} and IV_i^{Mkt} are the implied volatilities calculated from the model price and market price, respectively. Unlike the option price-based RMSE, IVRMSE is a standardized pricing error, as it avoids the high weight assigned to high price options. Table 4 provides the full summary of the sample pricing performance with decomposed details. Bold numbers indicate the minimum IVRMSE within each row.

[Insert Table 4 here]

For the overall performance, the total IVRMSE shows that models with realized measures generally perform better than those without realized measures. The GARV delivers the best fit (with a 21% IVRMSE reduction compared to HNG), followed by LHARG and RGARCH. The non-affine GARCH models (GARCH/GJR/NG/EG) are generally better than the affine GARCH models (HNG/HNGvd). A similar pattern is documented in Christoffersen et al. (2013). As the leverage effects are weak, there is no significant performance difference between symmetric (GARCH) and asymmetric models (GJR/NG).

For the decomposed performance, we isolate the 2015 subsample from the dataset and price it with parameters estimated from the full sample. The results clearly demonstrate that in the first year of SSE 50ETF options the behavior was significantly different than in the following year; the full sample parameters show extremely large IVRMSEs for the first year, but they drop sharply when the first-year data is eliminated. GARV remains the best model for all of the sub-cases and the log-linear models perform better when the volatility level is high and the option is deep out-of-money.

As the last three models rely on realized measures, we also provide performance comparisons for a range of realized measures. Table 5 reports the IVRMSE for the three models using different realized measures including the realized variance for different sampling frequency (RV1min, RV5min, RV10min, and RV30min) from Andersen and Bollerslev (1998), the two-scale realized variance (TSRV) from Zhang et al. (2005), the realized kernel (RK) of Barndorff-Nielsen et al. (2008), and the bi-power variation (BPV) of Barndorff-Nielsen (2004). The bold numbers indicate the minimum IVRMSE within each column. Interestingly, unlike the results based on volatility forecasting, the complex realized measures do not generally provide better option prices. Traditional realized variance with sampling intervals as short as 10 min provide reasonability good performances.

[Insert Table 5 here]

We also provide Black-Scholes (BS) results with volatility calibrated from the full sample. Given the dramatic changes in volatility levels between 2015 and 2018, it is not surprising that the BS delivers the worst pricing performance.

4.4 Out-of-sample Pricing Performance

To incorporate the realized measure, the number of parameters is significantly increased for the LHARG/GARV/RGARCH models. It is important to check that the superior performance of these models is not merely due to in-sample overfitting. In the literature, three major out-of-sample evaluation procedures have been proposed. The first method estimates the parameters using data from the first several years and then value option prices for the following years (e.g., Christoffersen and Jacobs (2004), etc.). The second method uses a rolling window framework in which the parameters are updated once in each time period (e.g., Christoffersen and Diebold (2006)). The third method splits the sample into Wednesday (for parameter estimation) and Thursday (for pricing evaluation) sub-samples within the same time period (e.g., Christoffersen et al. (2010)). For our study, as the Chinese data covers a much shorter period and is more volatile than the U.S. data, we use the rolling window as our primary method and splitting the sample as a robustness check[9].

The out-of-sample pricing performance evaluation is based on a rolling window of 252 trading days, with the parameters updated on a monthly basis. We evaluate the out-of-sample pricing errors from 2016/02 to 2018/02; the observations in from 2015/02 to 2016/01 are used as a pre-sample to determine the first parameter for the out-of-sample analysis. The results are presented in Table 6 with the decomposed results related to different moneyness, maturity, and VIX level.

[Insert Table 6 here]

Similar to the in-sample results, the models with realized measures have better out-of-sample pricing performance. The GARV model still generates the smallest total pricing error, but the decomposed results are mixed for the three models. The performance gain of the leverage GARCH models over the standard GARCH model is not significantly large. The HNGvd delivers results similar to the standard HNG model. In short, the results of the rolling window method of examining the out-of-sample data suggest that the performance gain of option pricing models based on realized measure are free from in-sample overfitting.

For cross validation method, we estimate the parameters using Monday/Wednesday/Friday data from the 2016/02 to 2018/02 period. Keeping the parameters fixed, we value the Tues-

[9]Given the much shorter sample, the splitting method in this study estimate parameters using Monday/Wednesday/Friday data and evaluate option prices using Tuesday/Thursday data. We use MWF/TTh to represent this method.

day/Thursday options within the same time span[10]. The results are presented in Table 7. Most of the results are the same as when the rolling window method is used.

[Insert Table 7 here]

5 Conclusions

This study tests a variety of discrete-time option pricing models with the SSE 50 ETF option data. We find that realized measure-based models have better in-sample and out-of-sample pricing performance for every kind of option. Unlike the results for volatility forecasting, complex realized measures do not provide better pricing results than simple ones like the conventional realized variance. The leverage effect is weak in the Chinese option market, which contradicts its documented effect in the U.S. market. The prices from the first trading year behave differently than in subsequent years, possibly due to the extreme market volatility and severe limited to arbitrage. This implies that it is necessary to analyze the data from this year separately when pricing Chinese options.

There are several issues that should be addressed in future studies. The first is the modeling of jumps in the underlying process. Christoffersen et al. (2012) highlighted the importance of dynamic jump intensities in option pricing. It is natural to expect that such a feature might also be important in the Chinese market. The second issue is the long-memory feature and the impact of different components of volatility. Christoffersen et al. (2008) provided a component GARCH model for option pricing. The last issue is the need for comparisons of discrete time models and continuous time models, especially for continuous time models with realized measures.

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Table 1: Summary of Competing Models

	G	GJR	NG	EG	HNG	HNGvd	RG	GARV	LHARG
Linear	✓	✓	✓		✓	✓		✓	✓
Log-Linear				✓			✓		
Affine					✓	✓		✓	✓
With RV							✓	✓	✓
With VRP						✓	✓	✓	✓
Close-form Pricing					✓	✓		✓	✓
# of Parameters	4	5	5	5	5	6	12	12	12

Table 2: Option Dataset Summary

	2015/02-2016/01	2016/02-2018/02	2015/02-2018/02
Total	3520	8761	12281
	(0.366)	(0.162)	(0.221)
Panel A: Partitioned by Moneyness			
S/K<0.95	839	818	1657
	(0.405)	(0.203)	(0.305)
0.95<S/K<1.05	2041	7162	9203
	(0.343)	(0.151)	(0.194)
1.05<S/K	640	781	1421
	(0.390)	(0.219)	(0.296)
Panel B: Partitioned by Maturity			
DTM<30	1672	3326	4998
	(0.374)	(0.159)	(0.231)
30<DTM<60	1507	3688	5195
	(0.360)	(0.164)	(0.221)
60<DTM	341	1747	2088
	(0.357)	(0.164)	(0.196)
Panel C: Partitioned by VIX Level			
VIX<15		3620	3620
		(0.121)	(0.121)
15<VIX<30	503	4473	4976
	(0.259)	(0.173)	(0.182)
30<VIX	3017	668	3685
	(0.384)	(0.308)	(0.370)

Note: The number of options in each category is provided. The average implied volatility is in parentheses.

Table 3: Full Sample Parameter Estimation: 2015/02-2018/02

	GARCH	GJR	NG	EG	HNG	HNGvd	GARV	RGARCH		LHARG
λ	-0.0353	0.2723	0.2057	0.0849	1.0950	1.1356	20.4439	0.0574	λ	6.6784
	(0.0361)	(0.0519)	(0.0299)	(0.0005)	(0.4612)	(0.8032)	(1.7763)	(0.0762)		(3.5198)
β	0.9080	0.9011	0.9161	0.9874	0.9867	0.9867	0.9818	0.9954	θ	8.20E-05
	(0.0027)	(0.0105)	(0.0114)	(0.0001)	(0.0010)	(0.0020)	(0.0011)	(0.0010)		(8.51E-06)
τ_1	0.0781	0.1299	0.0702	0.0354	7.91E-06	6.95E-06	2.07E-06	-0.0220	δ	1.4055
	(0.0027)	(0.0137)	(0.0113)	(0.0012)	(2.51E-07)	(5.12E-07)	(1.93E-07)	(0.0045)		(0.0879)
τ_2		-0.0708	-0.3372	0.1754	-23.8340	-25.4704	-61.4457	0.0460	$\theta\beta_d$	0.4928
		(0.0064)	(0.0442)	(0.0063)	(3.9761)	(4.1017)	(2.6017)	(0.0092)		(0.0547)
γ							0.3767	0.1085	$\theta\beta_w$	0.2648
							(0.0234)	(0.0217)		(0.0302)
ξ/κ							0.2105	2.2893	$\theta\beta_m$	0.0719
							(0.0244)	(0.2441)		(0.0068)
ϕ							2.89E-06	1.3024	$\theta\beta_q$	0.0494
							(2.70E-06)	(0.0291)		(0.0042)
d_1							8.14E-06	0.0032	$\theta\beta_y$	0.0580
							(2.22E-06)	(0.0092)		(0.0031)
d_2							371.1036	0.1924	α	2.35E-05
							(50.7164)	(0.0512)		(4.59E-06)
σ_u/ρ							0.0451	0.5817	γ	23.4299
							(0.0449)	(0.0483)		(7.3757)
$\chi/\tilde{\xi}$						1.0670	9.1939	0.0379	χ	356.6782
						(0.0537)	(1.8250)	(0.1472)		(559.7493)
$\log(\tilde{h})$	-8.5487	-7.4071	-7.7362	-7.7456	-7.5059	-7.5695	-7.8923	-8.2587	$\log(\tilde{h})$	-8.5758
	(0.0216)	(0.1134)	(0.0758)	(0.0210)	(0.0593)	(0.0639)	(0.0839)	(0.5922)		(0.5952)
π^P	0.9861	0.9956	0.9943	0.9874	0.9912	0.9912	0.9835	0.9954	π^P	0.9369
π^Q	0.9862	0.9870	0.9875	0.9874	0.9908	0.9908	0.9826	0.9954	π^Q	0.9999
ℓ	18800.2	19034.1	18820.5	19830.6	18655.6	18656.4	26618.3	19887.4	ℓ	269104

Note: Here we report $\theta\beta_i$ and $\theta\alpha_i$ instead of β_i and α_i for LHARG to make it easier to compare different models. The robust standard errors are in parentheses. π^P and π^Q are persistence parameters under physical and risk neutral measures, respectively. ℓ is the log-likelihood value.

Table 4: Full Sample Pricing Performance: 2015/02-2018/02

	BS	GARCH	GJR	NG	EG	HNG	HNGvd	LHARG	GARV	RGARCH
Total IVRMSE	12.2482	6.3602	6.2441	6.3489	5.8701	6.5301	6.5306	5.6821	**5.1600**	5.5718
Panel A: Partitioned by Time Period										
2015/02-2016/01	18.4495	10.3053	9.9942	10.2898	9.3670	10.9770	10.9758	9.2509	**8.3374**	8.7830
2016/02-2018/02	8.5751	3.7338	3.7949	3.7197	3.6002	3.3862	3.3896	3.2815	**3.0657**	3.5338
Panel B: Evaluation by Moneyness										
S/K<0.95	16.6198	8.1000	7.7831	8.0653	7.3916	9.4044	9.4021	7.4116	**6.7632**	7.0091
0.95<S/K<1.05	10.9419	5.7206	5.6223	5.6690	5.3168	5.7390	5.7392	5.1939	**4.7354**	5.0488
1.05<S/K	14.1084	7.9536	7.9874	8.2021	7.2333	7.4112	7.4166	6.3990	**5.6849**	6.8420
Panel C: Evaluation by Maturity										
DTM<30	13.5382	6.8011	6.6268	6.7727	6.4178	7.4022	7.4017	6.0932	**5.5587**	5.9368
30<DTM<60	11.8430	6.3989	6.2846	6.3709	5.7881	6.2062	6.2082	5.7734	**5.2115**	5.6348
60<DTM	9.7303	5.0033	5.0684	5.0951	4.5440	4.8767	4.8759	4.2307	**3.8886**	4.3753
Panel D: Evaluation by VIX Level										
VIX<15	10.3012	2.8344	2.7746	2.7710	2.8078	2.8551	2.8558	2.4064	**2.2971**	2.7943
15<VIX<30	6.3190	3.7128	3.7503	3.6475	3.5808	3.6011	3.6087	3.2686	**3.1925**	3.4713
30<VIX	18.4879	10.3817	10.1403	10.4008	9.4530	10.8107	10.8084	9.3333	**8.3533**	8.9051

Note: The bold numbers are the minimum IVRMSE values in each row.

Table 5: Pricing Performance Using Different Realized Measures

	LHARG	GARV	RGARCH
RV1min	5.8020	**5.0863**	5.6310
RV5min	**5.7282**	5.1600	5.5718
RV10min	5.9253	5.2245	**5.5281**
RV30min	5.9372	5.3567	5.5434
RK	5.9089	5.2841	5.6233
BV	5.8281	5.1562	5.5694
TSRV	5.9553	5.2847	5.6172

Note: This table reports the full sample pricing performances (IVRMSE) of three high frequency data-based option pricing models (GARV/LHARG/RGARCH) using different realized measures. We consider a variety of classes of estimators of asset price volatility, including the realized variance (RV1min, RV5min, RV10min, and RV30min) in Andersen and Bollerslev (1998), the two-scale realized variance (TSRV) in Zhang et al. (2005), the realized kernel (RK) in Barndorff-Nielsen et al. (2008), and the bi-power variation (BPV) in Barndorff-Nielsen (2004), The bold numbers are the minimum IVRMSE values in each column.

Table 6: Out-of-sample (Rolling Window) Pricing Performance: 2016/02-2018/02

	BS	GARCH	GJR	NG	EG	HNG	HNGvd	LHARG	GARV	RGARCH
Total IVRMSE	9.0341	4.1551	3.9833	3.9705	3.7481	3.9247	3.8531	3.3378	**3.2324**	3.3341
Panel A: Evaluation by Moneyness										
S/K<0.95	9.6641	4.1501	3.9875	3.7276	4.1867	4.2714	3.9163	**3.1152**	3.2558	3.1925
0.95<S/K<1.05	9.0475	4.0609	3.8939	3.8966	3.6008	3.8526	3.8267	3.2987	**3.2180**	3.3191
1.05<S/K	8.1892	5.0109	4.7927	4.8772	4.6104	4.2265	4.0376	3.9610	**3.3449**	3.6215
Panel B: Evaluation by Maturity										
DTM<30	9.3175	3.9309	3.8122	3.8930	3.6241	3.9890	3.9886	**3.1089**	3.2063	3.4035
30<DTM<60	9.3756	4.4317	4.2598	4.2098	3.9133	4.1514	4.0625	3.4520	**3.3850**	3.4985
60<DTM	7.6447	3.9566	3.6792	3.5689	3.6183	3.2519	3.0531	3.5131	2.9351	**2.8008**
Panel C: Evaluation by VIX Level										
VIX<15	5.8761	2.4747	2.4760	2.5273	2.5463	2.4859	2.4851	2.2456	**2.2443**	2.3770
15<VIX<30	7.4012	4.7196	4.4049	4.3748	3.7650	4.2332	4.0983	3.4616	3.3632	**3.2405**
30<VIX	12.8435	6.6072	6.6846	6.6018	7.3254	6.9726	6.9760	6.1652	**5.8145**	6.6765

Note: The out-of-sample pricing performance evaluation is based on a rolling window of 252 trading days, with the parameters updated on a monthly basis. We evaluate the out-of-sample pricing errors using observations from the 2016/02 to 2018/02 period, and use observations from the 2015/02-2016/01 period as a pre-sample to get the first parameter for the out-of-sample analysis. The bold numbers are the minimum IVRMSE values in each row.

Table 7: Out-of-sample Pricing Performance (MWF/TTh): in 2016/02-2018/02

	BS	GARCH	GJR	NG	EG	HNG	HNGvd	LHARG	GARV	RGARCH
Total IVRMSE	6.1244	3.0666	3.0691	3.0747	3.0794	3.1600	3.1526	**2.7796**	2.7805	2.8846
Panel A: Partitioned by Moneyness										
S/K<0.95	7.3434	2.7994	3.3350	2.7942	3.0241	3.9588	3.9483	**2.5132**	3.0451	3.0243
0.95<S/K<1.05	5.4585	2.8200	2.8669	2.8280	2.8371	2.9539	2.9489	**2.6092**	2.6207	2.7582
1.05<S/K	9.5211	5.2529	4.4278	5.3003	5.0030	4.0116	3.9996	4.3444	**3.8257**	3.8380
Panel B: Partitioned by Maturity										
DTM<30	5.9490	2.8301	2.8968	2.8324	2.9331	3.1090	3.0971	**2.7019**	2.8732	2.8812
30<DTM<60	6.5814	3.2018	3.1411	3.2015	3.1669	3.3095	3.2842	2.8546	**2.8085**	2.9243
60<DTM	5.4501	3.2461	3.2564	3.2838	3.1876	2.9360	2.9801	2.7777	**2.5149**	2.8063
Panel C: Partitioned by VIX Level										
VIX<15	4.9164	2.2558	2.2723	2.2553	2.3446	2.4316	2.4243	**1.9796**	2.0578	2.2127
15<VIX<30	4.3317	3.0044	3.0451	3.0062	2.9891	3.0626	3.0532	2.6264	**2.5910**	2.7669
30<VIX	15.2485	5.8826	5.7432	5.9286	5.8402	5.9633	5.9568	5.7518	5.7211	**5.5395**

Note: This table reports the out-of-sample pricing performance (IVRMSE) for the 2016/02 to 2018/02 period. We estimate the parameters using Monday/Wednesday/Friday data from the 2016/02 to 2018/02 period. Keeping the parameters fixed, we value the Tuesday/Thursday options within the same time period. The bold numbers indicate the minimum IVRMSE value in each row.

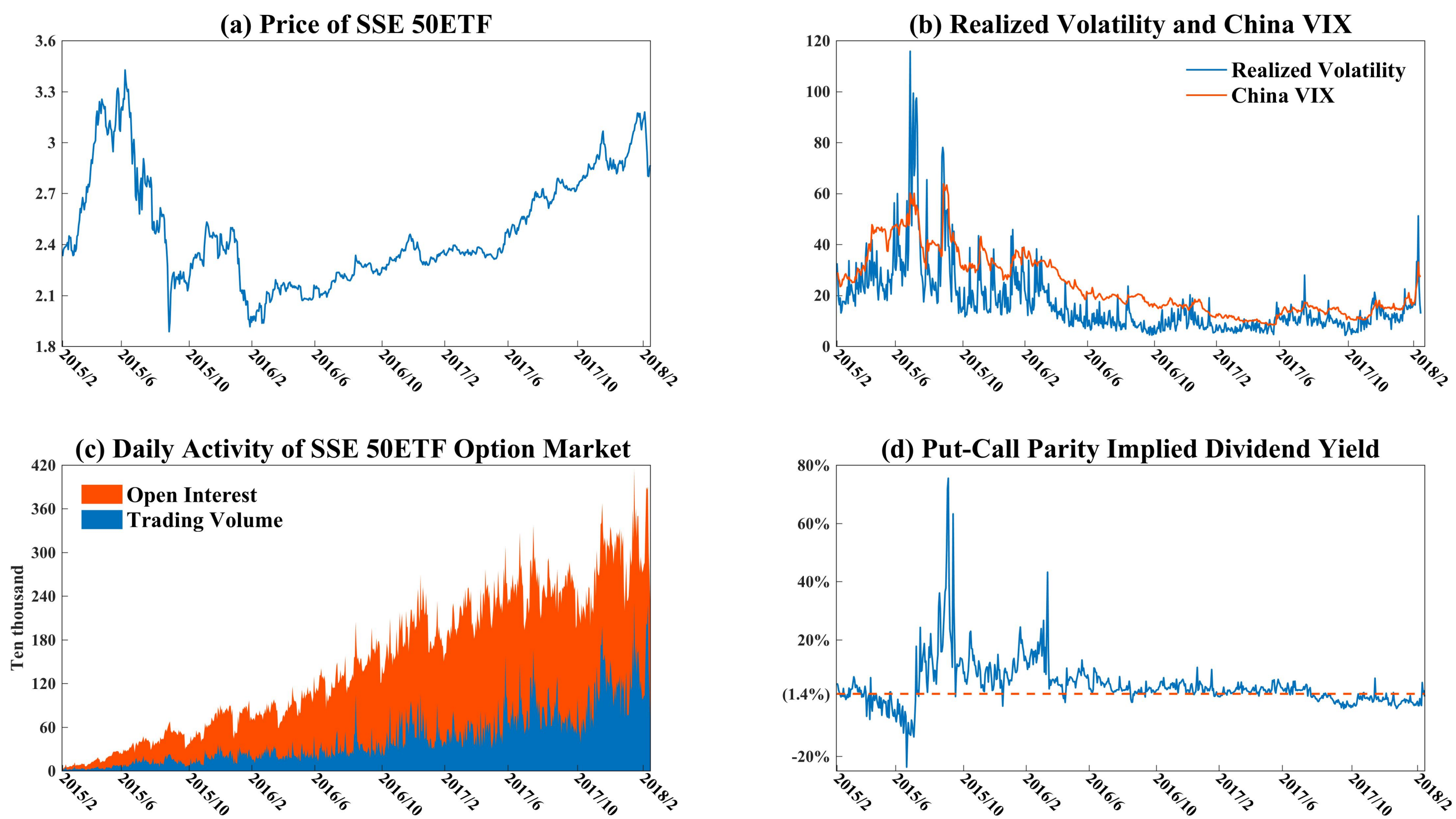

Figure 1: Time Series of SSE 50ETF

Note: Figure (a) shows the daily price of SSE 50ETF over the 2015/02 to 2018/02 sample period; Figure (b) reports the corresponding realized volatility (calculated using 5-minute returns) and China VIX (extracted from SSE 50ETF option prices and released by the Shanghai Stock Exchange); Figure (c) gives the trading volume and open interest (daily average). The blue solid line in Figure (d) is the daily average put-call parity implied dividend yield and the red dashed line is the actual historical average dividend yield during the sample period.