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1 The Introduction

In economics, there are basically two approaches to studying business cycles (or economic fluctuations). One approach is to regard business cycles as caused by exogenous shocks. In this view, the observed recurrent and irregular fluctuations are generated by repeated stochastic impulses to the economy. Historically, this approach can be traced back to Frisch (1933) and Slutsky (1937). Recent business cycle theories, such as real business cycles (RBC) and New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE), are often based on this approach.[1]

The other approach is to regard business cycles as systematic, deterministic, and self-generating recurrent cycles inherent to the underlying economy. The endogenous approach to business cycles is often found in the literature related to traditional Keynesian economics, nonlinear dynamics and disequilibrium economics.[2] The endogenous approach has not been well developed. Usually, the behavioral equations in the related models are not derived from optimization; the sources of economic fluctuation, although endogenous, are often unclear.

In this paper, we argue that whether business cycles are exogenous or endogenous depends on whether destabilizing mechanisms exist in the economy. Economists consider that the economy can be stabilized by price adjustment. While a price adjustment can be regarded as a stabilizing mechanism, there may be other mechanisms that can destabilize the economy. An important example, as illustrated in this paper, is the investment adjustment underlying Harrod's instability (or knife-edge) puzzle (Harrod, 1939).

Given the existence of a destabilizing mechanism, an economic fluctuation can be regarded as the interaction between these two mechanisms. If the stabilizing mechanism exerts a stronger force, the economy is inherently stable, and thus the driving force of economic fluctuation can only be exogenous. Conversely, if the destabilizing mechanism exerts a stronger force, the economy is inherently unstable, and the government will need to introduce a stabilization policy as an additional stabilizer.[3]

The paper is organized as follows. We first provide a brief introduction to the destabilizing mechanism in a simple Harrodian economy, and then show how economists, starting with Solow (1956), disregard this mechanism. Section 3 presents a simple statistical test to prove the existence of the destabilizing mechanism. In Section 4, we prove the existence of optimum capacity utiliza-

[1] The representative RBC literature includes Kydland and Prescott (1982), Long and Plosser (1983) and King et al. (1988a, 1988b), Christiano and Eichenbaum (1992), King and Rebelo (1999), among others. For a review of the RBC literature, see Rebelo (2005) and Gong and Semmler (2006). For the various NK-DSGE models, see Rotemberg and Woodford (1997), Gali (1999), Gertler et al. (1999), Woodford and Walsh (2005), Walsh (2003), Christiano et al. (2005), Smets and Wouters (2007) and Altig, et al. (2011), among others.

[2] The literature in this area includes, for example, Goodwin (1951), Benhabib and Nishimura (1979), Day (1982), Grandmont (1985), Flaschel et al. (2001, 2002), Gong and Lin (2008), among others.

[3] It should be noted that the stabilizing and destabilizing mechanisms do not exclude exogenous shocks as a possible source of economic fluctuation. However, exogenous shocks are no longer important under this approach.

tion, which is a key concept for investment adjustment. Section 5 presents our model, which includes both stabilizing and destabilizing mechanisms. Section 6 provides a stability analysis to the model without a government stabilization policy. Section 7 analyzes the model with government policy as an additional stabilizer. Finally, Section 8 provides the conclusion. The proof of the propositions is provided in the appendix.

2 What is a Destabilizing Mechanism and How is it Disregarded in Economics?

2.1 Harrod's Instability Puzzle

Ever since Harrod (1939) first introduce the concept, economists have engaged in an ongoing debate over destabilizing mechanisms. To a great extent, Harrod's instability puzzles motivated the research into dynamic economics, which includes not only business cycles, but also growth. The following equations can be considered as the structural form of the Harrodian economy:[4]

$$Y_t = C_t + I_t, \tag{1}$$

$$C_t = (1-s)Y_t, \tag{2}$$

$$K_t = (1-d)K_{t-1} + I_t, \tag{3}$$

$$Y_t^p = BK_{t-1}, \tag{4}$$

$$U_t = \frac{Y_t}{Y_t^p}, \tag{5}$$

$$\frac{I_t}{K_{t-1}} = \begin{cases} -\xi_0 + \xi U_{t-1}, & \text{if } U_{t-1} > \xi_0/\xi \\ 0, & \text{otherwise} \end{cases} \qquad \xi_0, \xi > 0 \tag{6}$$

In the above, Y_t is the output (or the aggregate demand); C_t is the consumption; I_t is the investment; K_t is the capital stock; Y_t^p is the potential output (or capacity); U_t is the capacity utilization; and $s \in (0,1)$, $B > 0$, and $d \in (0,1)$ are the saving ratio, the capital coefficient, and the depreciation rate, respectively.

The meanings of equations (1) - (6) are all straightforward. Equation (1) is the national income identity; equation (2) explains the determination of consumption; equation (3) describes the accumulation of capital stock; equations (4) and (5) are the definitions of potential output and capacity utilization;[5] and finally, equation (6) is the behavior function of investment.

The key equation that makes the system unstable is equation (6), the investment adjustment. The economic reasoning for this investment behavior is

[4] For references, see Sen (1970) and Gong (2001), among others.

[5] Note that by this definition, the capital stock in period $t-1$ is measured at the end of period $t-1$ (or at the beginning of period t) so that it provides the capacity for the production in period t. Of course one can also define $Y_t^p = BK_t$. This will not change the basic property of the model, but will increase the nonlinearity.

clear: investment creates capacity. If the demand for firm's output increases so that its existing capacity is insufficient, the firm will invest in creating more capacity. In this paper, we construct a dynamic optimization model that allows us to derive the investment function as in (6).

Note that equations (1) and (2) imply that

$$Y_t = \frac{1}{s} I_t \tag{7}$$

This reflects the method of determining output via a multiplier, as in traditional Keynesian economics, such as the IS-LM model. Thus, investment creates not only the capacity in the product market via (3) and (4), but also the demand via the multiplier $\frac{1}{s}$ in equation (7).

The market status is reflected in the comparison of Y_t and Y_t^p or in capacity utilization U_t. Assuming that the economy is in the range of $(\xi_0/\xi, +\infty)$, equations (4), (5) and (7) allow us to obtain

$$U_t = \frac{I_t}{sBK_{t-1}}$$

Substituting the investment function (6) into the above, we obtain

$$U_t = -\frac{\xi_0}{sB} + \frac{\xi}{sB} U_{t-1} \tag{8}$$

Denote $\bar{U}$ as the steady state of U_t. We find from the above that

$$\bar{U} = \frac{\xi_0}{\xi - sB}$$

For $\bar{U}$ to be positive and thus economically meaningful, we require that $\xi > sB$. However, this indicates that $\frac{\xi}{sB} > 1$. Therefore, the dynamic system as expressed in (8) is unstable. Figure 1 shows the trajectory of U_t.

Harrod called this instability the knife-edge problem. Intuitively, the problem can be expressed as follows. Suppose there is a certain degree of overheating in the economy. In this case, the investment will increase to meet the increased demand (or insufficiency in capacity). However, investment increases not only the capacity, but also the aggregate demand. As $\frac{\xi}{sB} > 1$, for the economy as a whole, the increased demand will be larger than the increased capacity. This will overheat the economy further and thus impel the firm to invest more. Conversely, if there is excess capacity, the firm will reduce its investment, but this reduction will also reduce the aggregate demand and thus the excess capacity will be enlarged.

2.2 Switching to Growth: Treatment by Solow (1956)

Harrod (1939) titled his seminal paper as "An Essay in Dynamic Theory." In modern economics, dynamics may refer either to the growth or to the business

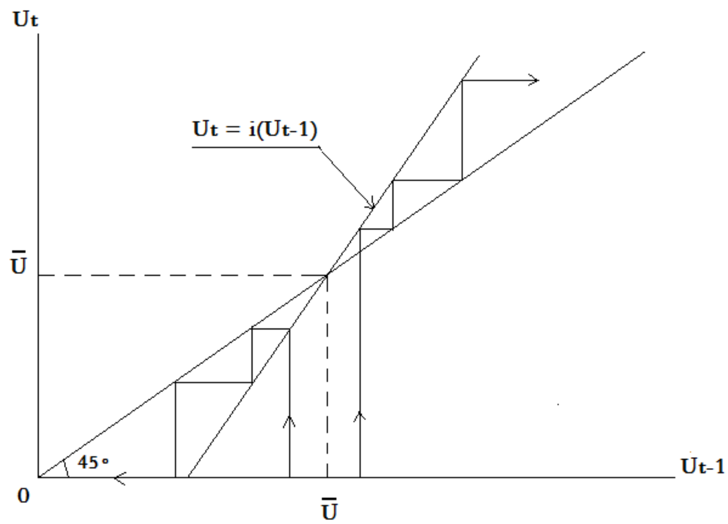

Figure 1: The Trajectory of U_t: $i(U_{t-1}) = -\frac{\xi_0}{sB} + \frac{\xi}{sB}U_{t-1}$

cycle. This seems to suggest that Harrod may not have confined his economic dynamics to the issue of growth; in fact, it was Solow (1956) who later restricted Harrod's dynamic problem to the growth field. Through this restriction, Solow resolved Harrod's puzzles within the neoclassical framework that ignores the demand side of an economy:

> A remarkable characteristic of the Harrod-Domar model is that it consistently studies long-run problems with the usual short-run tools. One usually thinks of the long run as the domain of neoclassical analysis, the land of the margin. Instead, Harrod and Domar talk of the long run in terms of a multiplier, an accelerator, "the" capital coefficient (Solow, 1956, pp.66)

Solow's neoclassical economy is very simple. Under the assumption of market clearing, the economy is composed of the first three equations (1) - (3) of Harrod's system plus the following production function:

$$Y_t = K_t^{1-\alpha}(A_t L_t)^{\alpha} \tag{9}$$

For Solow, this is an important improvement over Harrod's production technology because it allows the substitution between labor and capital. [6] Yet, by this

[6] In a communciation with one of the authors, Solow said that he did not believe that the knife-edge problem exists because the empirical economy does not behave like a knife-edge. Therefore, in his paper he does not try to deal with the knife-edge problem, but with Harrod's (1939) second problem, that is, the unbalanced growth between labor and the product market. This unbalanced growth problem is associated with the difference between the natural rate

transformation, the output is no longer determined by the demand via a multiplier as in (7), but by the supply via the production function (9). Although the system still permits equation (7) due to the inclusion of (1) and (2), the economic meaning of (7) is completely different. In a Harrodian economy, equation (7) explains the output determination in a demand-determined economy, which is the key hypothesis of Keynes (1936). Solow (1956), however, explains the method of investment determination. This indicates that the equation should be re-written as $I_t = sY_t$, while the output determination is given by (9).

We thus find that the investment behavior in Solow (1956) is simply a residue of national income identity. Investment does not respond to the market status of excess demand (or supply), as reflected by capacity utilization; it does not even create demand a via multiplier. Therefore, Harrod's destabilizing mechanism no longer exists.

2.3 Destabilizing Mechanisms in Recent Macroeconomics

Macro-dynamic models have been developed extensively since Solow (1956). New techniques such as dynamic optimization and log-linearization, among others, have been introduced. Investment theory has been enriched and provided with a micro-foundation, making investment no longer a simple residue.[7] Some Keynesian concepts, such as uncertainty, adjustment costs, and capital utilization, are also introduced. In particular, the adjustment cost that occurs with pricing generates the sticky pricing theory that is at the core of the NK-DSGE model.[8]

While all of these are important, the investment determination in these models still deviates from that of Harrod (1939). In most cases, it is the household rather than the firm that makes the investment decision. The household is assumed to own the capital stock and therefore makes the investment decision jointly with the consumption decision to maximize its utility (rather than profit). It is assumed that the firm rents the capital stock period by period. This institutional arrangement is based on the assumption that the capital stock is homogeneous. The RBC and most of the NK-DSGE models, such as Christiano et al. (2005), all make this assumption.

The assumption of homogeneous capital is certainly unsatisfactory. Recently, a few attempts have been made to introduce firm-specific capital, and thus it is the firm rather than the household that decides on investment.[9]

Yet, in none of these treatments, does the investment respond to the market demand relative to the capacity (or capacity utilization), as asserted by Harrod (1939). For instance, although capital utilization (a similar concept to capacity

of growth in the labor market and the warrant rate of growth in the product market. To resolve this problem, the Cobb-Dauglass production function must be introduced to allow the substitution between labor and capital.

[7] See, for instance, Abel and Eberly (1994), among others.

[8] For the typical sticky pricing models, see, for instance, Rotemberg (1982), Calvo (1983), Mankiw (1985), and Taylor (1999), among others.

[9] For research on firm-specific capital, we refer the readers to Woodford (2005), Sveen and Weinke (2007), and Atig et al. (2011), among others.

utilization in the Harrodian economy) has been introduced, so that (9) may become $Y_t = (U_t K_t)^{1-\alpha}(A_t L_t)^{\alpha}$, the capital utilization U_t is set by the firm (or even by the household in a model with homogeneous capital). This indicates that it is still the firm's willingness to supply (reflected by the firm's optimum choice) that determines the output. This differs from the models of Harrod and also the original model of Keynes (1936), in which the firm's production output is determined by the demand, which is exogenous to the firm and thus not controllable (or set) by the firm. Given that the firm's output is exogenous, the firm's capacity utilization U_t should also be exogenous and also not controllable (or set) by the firm. The investment adjustment in response to capacity utilization (which is exogenous to the firm) is the key hypothesis for the investment to be a destabilizing mechanism, thus we find no destabilizing mechanism in the models.[10]

Although most macroeconomic models do not involve the destabilizing mechanism, a few models do allow for Harrod's investment adjustment and thus entail a destabilizing mechanism. In these models, investment is often in response to capacity utilization, which generates the instability, as in Harrod (1939). To overcome the instability, stabilizing mechanisms such as pricing and government stabilization policies are introduced. This type of model also has very rich empirical results for both developed and developing economies.[11] However, the behavior functions in these models are not usually derived by optimization.

In this paper, we will discuss the business cycles in terms of this approach, while providing the optimization for deriving the behavior functions underlining the model. We find that this approach enriches our understanding of economic fluctuation. It also allows us to identify the role of government stabilization policies. First, we shall provide a simple test to empirically demonstrate the existence of the destabilizing mechanism.

3 Empirical Existence of a Destabilizing Mechanism

One of the arguments that Solow makes for disregarding the knife-edge puzzle is that the empirical economy does not behave like a knife edge.[12] It is true that empirically, economies do not usually appear to be divergent as in Figure 1. Instead, they move cyclically around some steady state. The impulse-response function also shows that if there is a deviation from the steady state, e.g. due to a shock, an economy will move back to the steady state after some time. This seems to suggest that the economy is inherently stable and the economic

[10] Later in this paper, we demonstrate that it is possible to establish a method for determing output from demand (i.e., exogenous to the firm) in an optimization model, which can thus be understood as bounded rationality.

[11] This type of models is applied to the U.S. economy in Flaschel et al. (2001), to the German economy in Flaschel et al. (2002), and to a developing economy such as China in Gong and Lin (2008) and Gong (2013).

[12] This is discussed in a letter written by Solow to one of the authors of this paper.

fluctuations that we observe empirically can only be caused by exogenous shocks.

However, what we observe empirically is the regular enforcement of government stabilization policies. An interesting question then is if without government stabilization policies, an economy can still be stabilized? The answer to that question allows us to see whether a destabilizing mechanism exists empirically. Next, we shall provide a simple statistical test to answer this question.

We use annual data from China because the stabilization policy introduced in this paper is consistent with the current Chinese economy, and all of the parameters used in the simulation are estimated according to these annual data.[13] We consider four economic variables, real GDP, real investment, CPI, and money supply, denoted as Y_t, I_t, P_t and M_t respectively. Note that the money supply M_t reflects the monetary policy in the current Chinese economy.[14] All variables are detrended using an HP filter.

As a test, we first estimate a simple VAR model,

$$\begin{bmatrix} \hat{Y}_t \\ \hat{I}_t \\ \hat{P}_t \\ \hat{M}_t \end{bmatrix} = \begin{bmatrix} b_{01} \\ b_{02} \\ b_{03} \\ b_{04} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \hat{Y}_{t-1} \\ \hat{I}_{t-1} \\ \hat{P}_{t-1} \\ \hat{M}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{bmatrix} \tag{10}$$

where a variable $\hat{X}_t$ is the detrended variable X_t, and $\varepsilon_{i,t}$, $i = 1, 2, 3, 4$, is an i.i.d. innovation. The impulse response functions due to a shock in investment (5% deviation from the steady state) are shown in Figure 2. As the figure shows, all of the responses eventually return to the steady state. Therefore the economy is assumed to be stable.

However, the stability may be due to the stabilization policy M_t. To examine whether the economy can remain stable without stabilization policy, M_t, we need to set up $\hat{M}_t$ and $\hat{M}_{t-1}$ in the model (10) to 0. Thus, the model is transformed to

$$\begin{bmatrix} Y_t \\ I_t \\ P_t \end{bmatrix} = \begin{bmatrix} \hat{b}_{01} \\ \hat{b}_{02} \\ \hat{b}_{03} \end{bmatrix} + \begin{bmatrix} \hat{b}_{11} & \hat{b}_{12} & \hat{b}_{13} \\ \hat{b}_{21} & \hat{b}_{22} & \hat{b}_{23} \\ \hat{b}_{31} & \hat{b}_{32} & \hat{b}_{33} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ I_{t-1} \\ P_{t-1} \end{bmatrix} \tag{11}$$

where $\hat{b}_{ij}$ is the estimated b_{ij} from the model (10). The impulse response to the same investment shock, but now computed from (11), is shown in Figure 3. As the figure shows, the responses do not return to the steady state and thus the economy is no longer stable. This indicates that, at least in the Chinese economy, there must be some destabilizing mechanism that exerts a greater force than that exerted by pricing mechanism.

[13] Quarterly data from China were not available until recently.

[14] See the discussion in Gong and Lin (2008) and the discussion in Section 4 of this paper,

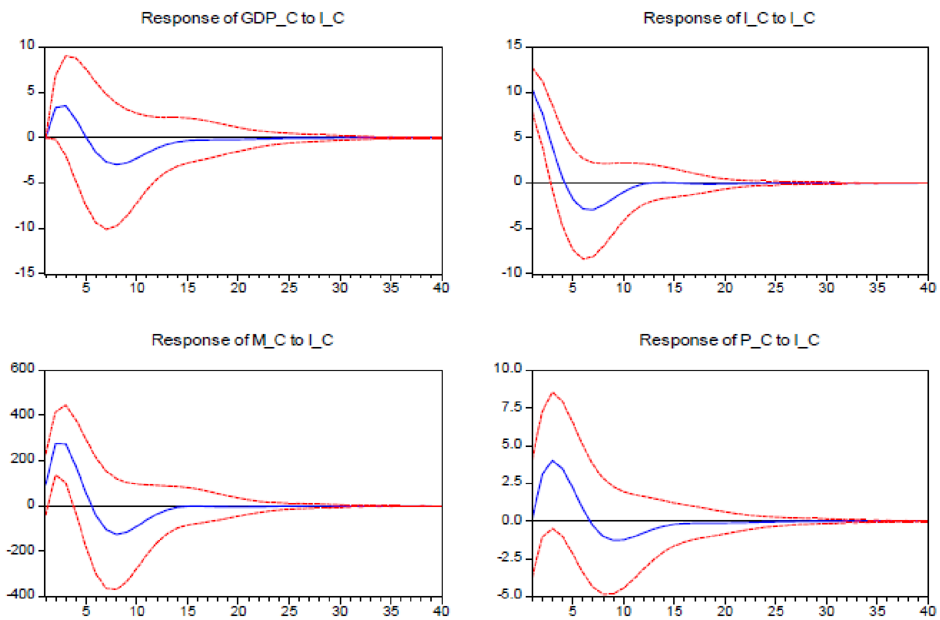

Figure 2: The Impulse Response Functionn with Monetary Policy: Response to Investment Shock

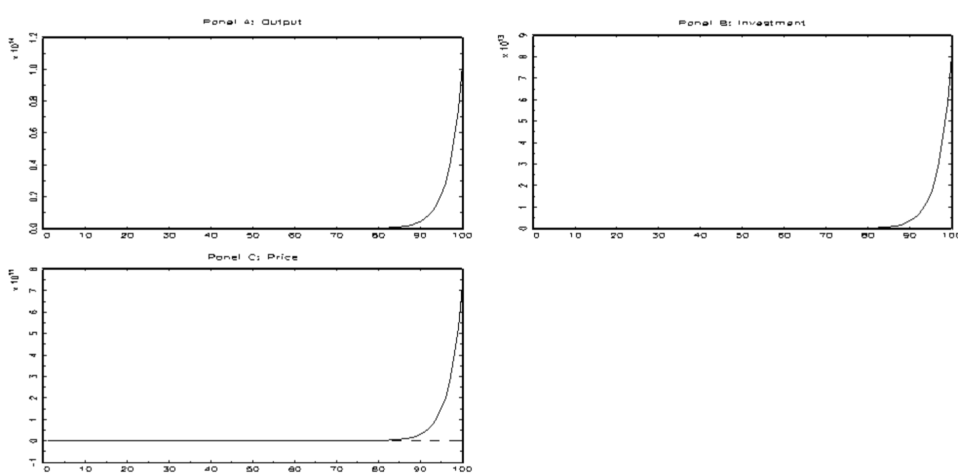

Figure 3: Impulse Response Function without Monetary Policy: Response to Investment Shock.

4 The Existence of Optimum Capacity Utilization

As shown in the Harrodian economy, capacity utilization is a key concept for investment adjustment and thus for the existence of a destabilizing mechanism. In this section, we first prove that under certain conditions, there exists a time-invariant optimum level of capacity utilization. This is important because given the optimum level of capacity utilization, investment can be understood as an adjustment to that optimum.

4.1 Technology

We begin our discussion with the production technology used in this paper. One criticism of the Harrodian economy concerns the production technology adopted by Harrod (1939), which assumes no substitution between capital and labor. We illustrate that even if we allow for the substitution between capital and labor in the production function, Harrod's instability theory still holds.

For a typical firm $j \in [0, 1]$ in period t, the production technology (or the input-output relation) is assumed to be in the form of the Cobb-Dauglass function, described as follows:

$$Y_{j,t} = a\left(A_{j,t}L_{j,t}\right)^{\alpha} K_{j,t-1}^{1-\alpha} \tag{12}$$

where, $Y_{j,t}$ is the output produced for j at t; $K_{j,t-1}$ is the capital stock specific to j measured at the end of period $t-1$, so it provides the production facility in period t; $L_{j,t}$ is the labor employed by j; $A_{j,t}$ is a measure of labor efficiency, whose dynamics reflects technical progress; and a is a time-invariant parameter.

This production function is very common in the literature. Here, we use $K_{j,t-1}$ rather than $K_{j,t}$ to enter the production function, merely to emphasize that the capital stock in period t is fixed. Given a fixed capital stock, the capacity utilization, a key concept in Harrod (1939), can naturally be derived.

4.2 Capacity Utilization

The input-output relation expressed in (12) implies that

$$\begin{aligned} A_{j,t}L_{j,t} &= \left(\frac{Y_{j,t}}{aK_{j,t-1}^{1-\alpha}}\right)^{1/\alpha} \\ &= Y_{j,t}\left(\frac{Y_{j,t}}{K_{j,t-1}}\right)^{\frac{1-\alpha}{\alpha}} \frac{1}{a^{1/\alpha}} \end{aligned}$$

Define $B^{\frac{1-\alpha}{\alpha}} \equiv a^{1/\alpha}$. Thus, we find from the above that

$$L_{j,t} = \frac{Y_{j,t}}{A_{j,t}}\left(\frac{Y_{j,t}}{BK_{j,t-1}}\right)^{\frac{1-\alpha}{\alpha}} \tag{13}$$

Equation (13) can be understood as the demand function for labor, given the firm's production output $Y_{j,t}$, the capital stock $K_{j,t-1}$ and the labor efficiency $A_{j,t}$.

As in Harrod (1939), we define the capacity utilization $U_{j,t}$ as

$$U_{j,t} \equiv \frac{Y_{j,t}}{BK_{j,t-1}} \tag{14}$$

Further expressions might be needed here to capture the economic meaning of capacity utilization $U_{j,t}$ under the Cobb-Dauglass production function that allows substitution between capital and labor. Suppose that in period t, the production facility represented by $K_{j,t-1}$ is given. The production activity can thus be understood as employing laborers to run the facility: the longer the facility runs, the larger the output produced. Therefore, we can define the capacity utilization $U_{j,t}$ as in (14), which roughly reflects the proportion of time that the facility runs in period t (which generates output $Y_{j,t}$) over the normal working time available in a period (which generates the potential output $BK_{j,t-1}$)[15]

Now, substituting (14) into (13), we obtain

$$L_{j,t} = \frac{Y_{j,t}}{A_{j,t}} \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}} \tag{15}$$

Equation (15) simply states that the demand for labor $L_{j,t}$ is used to run the facility to produce the output $Y_{j,t}$. Thus, $L_{j,t}$ is positively determined by output $Y_{j,t}$, and negatively by labor efficiency $A_{j,t}$, and is adjusted by capacity utilization $U_{j,t}$. There is no doubt that the longer the facility runs in a given period (or the higher the capacity utilization), the more labor is needed to produce additional output.

4.3 Cost Function

Given the definition of capacity utilization $U_{j,t}$ as expressed in (14), we find that the production cost can also be understood as a function of $U_{j,t}$. Let $C_{j,t}$ denote the total cost in real terms for firm j at t, and let $W_{j,t}$ denote the real wage rate paid by the firm. Ignoring the other intermediate input (such as raw materials, etc.), the total cost of the firm can be written as

$$C_{j,t} = L_{j,t}W_{j,t} + vK_{j,t-1}$$

where the labor cost $L_{j,t}W_{j,t}$ can be regarded as a variable cost (because, as shown below, it will vary with the produced output $Y_{j,t}$), and $vK_{j,t-1}$ is a fixed cost, which will not vary with output $Y_{j,t}$ but with the capital stock $K_{j,t-1}$. Now, expressing $L_{j,t}$ in terms of (15), we obtain

[15] For instance, we can assume that when the capacity utilization $U_{j,t}$ is equal to 1, the facility runs for 40 hours a week. Given this capacity utilization, and the capital stock $K_{j,t-1}$ and the output $Y_{j,t}$ produced in 40 hours, we can always find a value of B, the capital coefficient, that makes $Y_{j,t}/(BK_{j,t-1})$ equal 1.

$$C_{j,t} = \frac{W_{j,t}Y_{j,t}}{A_{j,t}} \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}} + vK_{j,t-1} \tag{16}$$

Suppose $\frac{W_{j,t}}{A_{j,t}} = \omega$, that is, the real wage $W_{j,t}$ increases at the same rate as the labor efficiency $A_{j,t}$.[16] Thus, given the total cost as in (16), the marginal cost $C_{j,t}^{'} \equiv \frac{\partial C_{j,t}}{\partial Y_{j,t}}$ and the average cost $c_{j,t} \equiv \frac{C_{j,t}}{Y_{j,t}}$ can be written as

$$C_{j,t}^{'} = \frac{\omega}{\alpha} \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}} \tag{17}$$

$$c_{j,t} = \omega \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}} + \frac{v}{B} \left(U_{j,t}\right)^{-1} \tag{18}$$

Above, $\omega \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}}$ can be regarded as the average variable cost (AVC) and $\frac{v}{B} \left(U_{j,t}\right)^{-1}$ as the average fixed cost (AFC).

4.4 Optimum Capacity Utilization when Capital Stock is not Adjustable

It is useful to derive the level of capacity utilization that minimizes the average cost. From (18), the first-order condition for this minimization problem can be written as

$$\frac{1-\alpha}{\alpha}\omega \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}-1} - \frac{v}{B} \left(U_{j,t}\right)^{-2} = 0$$

Solving the above equation for $U_{j,t}$, we obtain

$$U_{j,t}^{*} = \left[\frac{\alpha v}{\omega \left(1-\alpha\right) B}\right]^{\alpha} \tag{19}$$

This can be regarded as the optimum capacity utilization when the capital stock is given (or not adjustable). We find that it is indeed time-invariant.

Let $C_{j,t}^{'} = c_{j,t}$. From (17) and (18), we find that

$$\frac{\omega}{\alpha} \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}} = \omega \left(U_{j,t}\right)^{\frac{1-\alpha}{\alpha}} + \frac{v}{B} \left(U_{j,t}\right)^{-1}$$

Solving the above equation for $U_{j,t}$, we again obtain (19). Therefore, the level of capacity utilization that minimizes the average cost, as expressed in (19), is also the level at which the marginal cost cuts the average cost. Figure 4 provides the different costs as a function of capacity utilization.

The above discussion seems to suggest that the standard firm theory in microeconomics with regard to the cost function still holds in terms of capacity utilization. In particular, given the capital stock $K_{j,t-1}$, the marginal cost and the variety of average costs can all be expressed as a function of capacity utilization $U_{j,t}$. In addition, if $\frac{W_{j,t}}{A_{j,t}} = \omega$, the functions are also time-invariant. This also indicates that the optimum level of capacity utilization that minimizes the average cost can be a constant (see equation (19)).

[16] This assumption seems plausible in developed economies where surplus labor is not significant. Meanwhile it will make our model much simple since we do not need to introduce the labor market into our model.

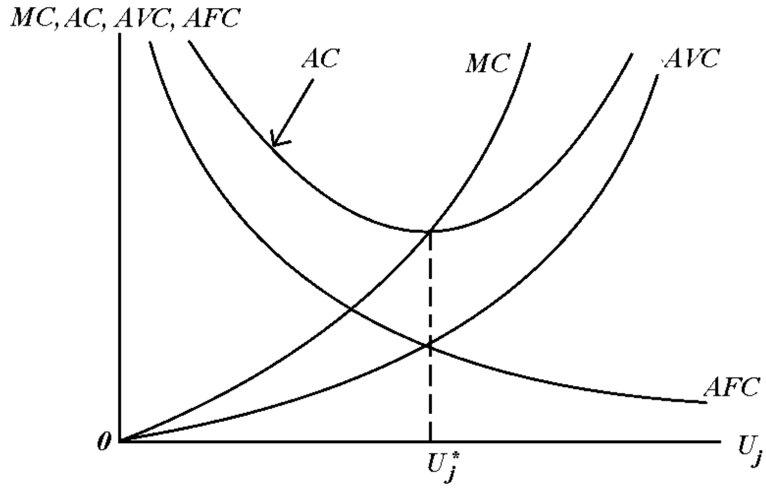

Figure 4: Marginal cost (MC), average cost (AV), average variable cost (AVC) and average fixed cost (AFC) as functions of capacity utilization U_j

4.5 Optimum Capacity Utilization when Capital Stock is Adjustable

The capacity utilization expressed in (19) can be understood as the optimum capacity utilization when capital stock is given (not adjustable). Suppose now that the average cost is too high due to the high capacity utilization. In this case, investment is needed to expand the capacity to reduce the average cost.

Investment is constructed for future capacity; therefore, we assume that the firm has been given a sequence of expected demands $E\{Y_{j,t+k}\}_{k=0}^{\infty}$, a sequence of technologies $E\{A_{j,t+k}\}_{k=0}^{\infty}$, and a sequence of real wages $E\{W_{j,t+k}\}_{k=0}^{\infty}$, among others, when making an investment decision in period t. The investment decision problem can thus be expressed as the choice of a sequence of investments $\{I_{j,t+k}\}_{j=0}^{\infty}$ such that

$$\max_{\{I_{j,t+k}\}_{k=0}^{\infty}} E\sum_{k=0}^{\infty}\beta^k\left[P_{j,t+k}Y_{j,t+k} - P_{t+k}c_{j,t+k}Y_{j,t+k} - (1+r)P_{t+k}I_{j,t+k}\right] \quad (20)$$

subject to

$$K_{j,t+k} = (1-d_j)K_{j,t+k-1} + I_{j,t+k} \quad (21)$$

where E is the expectation operator; β is the discount factor; r can be regarded as the interest rate that reflects the firm's opportunity cost of investment; $P_{j,t+k}$ is the price of the product produced by firm j; P_{t+k} is the aggregate price level; $c_{j,t+k}$ is the average cost, expressed by (18); and d_j is the depreciation rate. Equation (21) can be regarded as the process of capital accumulation.

Proposition 1 provides the solution to this optimization problem.

Proposition 1 *Suppose* $E\left[\frac{W_{j,t+k}}{A_{j,t+k}}\right] = \omega$ *and* $E\left[\frac{P_{t+k}}{P_{t+k-1}}\right] = \pi$*. Then, the problem in (20) - (21) with* $c_{j,t+k}$*, given by (18), allows us to obtain*

$$U^*_{j,t+k} = U^*_j = \left(\frac{\alpha\left[(1+r)/(\beta\pi) - (1+r)(1-d_j) + v_j\right]}{(1-\alpha)B\omega}\right)^{\alpha} \tag{22}$$

where $k = 1, 2, 3, ...$

The proof of this proposition is given in the appendix.

Equation (22) is quite similar to equation (19) when the capital stock is given (or not adjustable). Consider $\beta = 1$, $r = 0$, $\pi = 1$, and $d_j = 0$, so that we return to a one-period decision. In this case, the two equations (22) and (19) coincide. This comparison allows us to obtain the following interpretation of the investment decision

> Suppose the investment is divisible. Whatever the level of expected demand, $E[Y_{j,t+1}]$, the purpose of investment $I_{j,t}$ is simply to adjust the capital stock $K_{j,t}$ to the level at which $E[Y_{j,t+1}]/(BK_{j,t})$ is equal to U^*_j, as in (22), which minimizes the average cost of production.

5 The Model

Given the technology, the variety of cost functions, and the existence of optimum capacity utilization as described above, we now discuss the decisions made by firm j. There are three types of decisions to be made: production, pricing and investment. One of the important assumptions that differentiates the model from most other models is that the firm make its decisions separately and sequentially. The sequence of decision events pertaining to j is as follows.

At the very beginning of period t, the firm makes its investment plan $I^*_{j,t}$. This is essential because there is a time interval in which the firm must build $K_{j,t}$ via investment to serve the capacity for period $t+1$. At the same time, the monetary policy for period t is also announced and executed, which may constrain the firm's optimum plan $I^*_{j,t}$. This results in the firm's actual investment $I_{j,t}$ in the period. Next, the firm sets its price $P_{j,t}$, and announces it to the public. Given the announced price $P_{j,t}$, the demand for the firm's product $Y_{j,t}$ is realized. The firm then purchases labor to satisfy the demand for its output $Y_{j,t}$. This can be regarded as the firm's production decision.

Next, we follow this event sequence to construct our model.

5.1 Investment without Financial Constraint

Given an optimum level of capacity utilization U^*_j, as expressed in (22), we shall now consider how the investment should be made. Because the investment carried in period t creates the capital stock $K_{j,t}$ that serves the capacity for period $t+1$, the optimum investment, denoted as $I^*_{j,t}$, should satisfy

$$\frac{E\left[Y_{j,t+1}\right]}{B\left[(1-d_j)K_{j,t-1}+I^*_{j,t}\right]}=U^*_j$$

where the left side of the equation can be understood as the expected capacity utilization for period $t+1$. Resolving this equation for $I^*_{j,t}$, we obtain

$$I^*_{j,t}=\frac{E\left[Y_{j,t+1}\right]}{BU^*_j}-(1-d_j)K_{j,t-1} \tag{23}$$

Dividing both sides by $K_{j,t-1}$, we obtain

$$\begin{aligned}\frac{I^*_{j,t}}{K_{j,t-1}} &= \frac{E\left[Y_{j,t+1}\right]}{BU^*_jK_{j,t-1}}-(1-d_j)\\ &= -(1-d_j)+\left(\frac{E[y_{j,t+1}]}{U^*_j}\right)U_{j,t} \end{aligned} \tag{24}$$

where $E[y_{j,t+1}]$ is the expected gross growth rate of product j.

Note that in the discussion with regard to the event sequence, we assume that the investment decision is made at the very beginning of period t. This indicates that the firm may not observe its market demand in t and thus the capacity utilization $U_{j,t}$. In this case, $U_{j,t}$ in (24) can simply be regarded as the expected capacity utilization of $U_{j,t}$ given the information in $U_{j,t-1}$. Suppose that $E\left[U_{j,t}\right]=U_{j,t-1}$ and $E[y_{j,t+1}]=y_j$.[17] We find that (24) can be re-written as

$$\frac{I^*_{j,t}}{K_{j,t-1}}=-(1-d_j)+\frac{y_j}{U^*_j}U_{j,t-1} \tag{25}$$

This equation indicates that the investment rate $\frac{I^*_{j,t}}{K_{j,t-1}}$ depends on the observed capacity utilization: the higher the capacity utilization, the higher the investment rate.

5.2 Monetary Policy

Usually, a monetary authority has two approaches available to manage demand (or stabilization). The first is to target, or exogenize, the money supply. In this case, the interest rate could be endogenized. This method of demand management is introduced in the traditional IS-LM model, and was practiced by most developed countries before 1990s. It was also introduced by the current monetary authority in China.[18] The second approach is to target interest rates.

[17] This assumption will make our investment behavior the same as in the Harrodian economy (see equation 6),

[18] See Gong (2012)

In this case, the money supply is endogenized while the interest rate is exogenous. This occurred recently in the U.S. and other developed countries. The major reason for this transition, i.e., from targeting money supplies to targeting interest rates, is the institutional change in financial markets.[19]

Following Gong and Lin (2008) and Gong (2013), in this paper we use the money supply as the major target for the monetary authority to conduct its demand management. Let p^* denote the gross inflation rate that the monetary authority wants to target. Thus, the monetary policy rule can be formulated as

$$m_t - m_{t-1} = \kappa_p(p^* - p_{t-1}) + \kappa_m(m^* - m_{t-1}), \quad \kappa_m, \kappa_p > 0 \tag{26}$$

where m_t and m^* represent the actual and targeted gross growth rate of the money supply. This formulation indicates that the money supply will change in response to whether inflation and the money supply in the last period were below or above their targets.

5.3 Investment with Financial Constraint

Next, we consider the effects of monetary policy on the economy. It is apparent that it should affect investments. The investment that we consider is the optimum investment desired by firm j if there is no financial restriction. A financial restriction may affect this investment through two channels: the interest rate and the credit supply. Consistent with the money supply rule adopted in (26), we consider the credit supply.

Suppose that our representative firm j is able to acquire a loan from a commercial bank (in real terms) up to $\Delta M_{j,t-1}$ for its investment.[20] This indicates that the firm's investment under credit constraint can be written as

$$I_{j,t} = \begin{cases} I^*_{j,t} & I^*_{j,t} < \Delta M_{j,t-1} \\ \Delta M_{j,t-1} & \text{otherwise} \end{cases} \tag{27}$$

Let ΔM_{t-1} denote the total additional money (or credit) from the commercial bank system in period $t-1$. This money supply is targeted by the monetary authority, and it is the amount of money that a commercial bank can lend to finance the investment. Given ΔM_{t-1}, we write $\Delta M_{j,t-1}$ as

$$\Delta M_{j,t-1} = l_j \Delta M_{t-1} \tag{28}$$

where $l_j \in [0, 1)$ is the proportion of total credit allocated to j.

Under this credit plan, the firm makes its investment decisions according to (27). Summing all $I_{j,t}$'s, we get the aggregate investment I_t:

$$I_t = \int_0^1 I_{j,t} dj \tag{29}$$

[19] For a discussion of this transition, see Taylor (1993).

[20] Here, we assume that it is the money supply (or credit) in period $t-1$ that is used for financing the investment in period t.

Depending on the credit ratio l_j assigned to the firm, we find that for some j's, investments are bounded, that is, $I_{j,t} = \Delta M_{j,t-1}$; whereas for others, investments are at the optimum, that is, $I_{j,t} = I^*_{j,t}$, where $I^*_{j,t}$ is given by (25). Re-arranging the index of the firms such that the first n_1 proportion of the firms are bounded, we can write (29) as

$$I_t = \phi \Delta M_{t-1} + \int_{n_1}^{1} \left[-(1-d_j)K_{j,t-1} + \left(\frac{1+y_j}{U_j^*} \right) U_{j,t-1} K_{j,t-1} \right] dj \tag{30}$$

where $\phi = \int_0^{n_1} l_j dj$. Under the identical assumption of a representative agent, the above equation can be re-written as

$$I_t = \phi \Delta M_{t-1} - (1-d) \int_{n_1}^{1} K_{t-1} dj + \frac{1+y}{U^*} U_{t-1} \int_{n_1}^{1} K_{t-1} dj$$

Dividing both sides of the above equation by K_{t-1}, the aggregate capital stock, we obtain from the above

$$\frac{I_t}{K_{t-1}} = \phi \frac{\Delta M_{t-1}}{K_{t-1}} - (1-d)n_k + \frac{(1+y)n_k}{U^*} U_{t-1} \tag{31}$$

where n_k can be regarded as the proportions of capital stock from those unrestricted firms:

$$n_k \equiv \frac{\int_{n_1}^{1} K_{j,t-1} dj}{K_{t-1}}$$

Here, we assume this proportion to be time-invariant.

To ensure our analysis is tractable, we assume a linear relationship between the aggregate money supply and the aggregate capital stock $K_{t-1} = \eta M_{t-2}$. As we are working with aggregate variables, the rationality of this linear relationship is considered more from a statistical view point.[21]

Given this linear proposition, we can now re-write our aggregate investment function (31) as

$$\frac{I_t}{K_{t-1}} = -\xi_i + \xi_u U_{t-1} + \xi_m (m_{t-1} - p_{t-1}) \tag{32}$$

where $m_{t-1} - p_{t-1} \approx M_{t-1}/M_{t-2}$ is the approximate gross growth rate of the credit supply in real terms. Note that here, m_t is the nominal growth rate of the credit supply, which is targeted by the monetary authority. The parameters ξ_i, ξ_u and ξ_m are given by

$$\xi_i = (1-d)n_k, \quad \xi_u = \frac{(1+y)n_k}{U^*}, \quad \xi_m = \phi/\eta.$$

[21] Gong and Lin (2008) and Gong (2013) estimate the investment function using data from China and show that the estimation is statistically significant.

5.4 Pricing

As noted above, the price adjustment is a stabilization mechanism. Yet, if we follow New Keynesian economics as in this paper, the price adjustment is also sticky. This indicates that the force exerted by the price mechanism may not be large enough to overcome the force exerted by the destabilization mechanism in the economy.

Following the sticky price theory of New Keynesian economics, the probability of firm j re-optimizing its price in period t is $1-\theta$. Otherwise (with a probability of θ), it simply indexes the price as $\tilde{P}_{j,t}$ according to the rule:

$$\tilde{P}_{j,t} = p_{t-1} P_{j,t-1} \tag{33}$$

where $p_{t-1} \equiv \frac{P_{t-1}}{P_{t-2}}$ is the gross inflation rate observed in period $t-1$.

If firm j has a chance to re-optimize its price, the following optimization problem is constructed to derive the optimal price $P^*_{j,t}$:

$$\max_{P^*_{j,t}} E \sum_{k=0}^{\infty} \beta^k \theta^k \left(\pi^k P^*_{j,t} Y_{j,t+k} - P_{t+k} c_{j,t+k} Y_{j,t+k} \right) \tag{34}$$

subject to

$$Y_{j,t+k} = \left(\frac{P_{t+k}}{\pi^k P^*_{j,t}} \right)^{\epsilon} Y_{t+k} \tag{35}$$

Equation (34) indicates that the firm expects that in the future, at $t+k$, the price will be indexed to $\pi^k P^*_{j,t}$ if it has no opportunity to re-optimize its price after setting $P^*_{j,t}$. The probability of this condition occurring is equal to θ^k. Equation (35) can be regarded as the demand function under this condition, for product j at the future time $t+k$; and $c_{j,t+k}$ is the average cost (as expressed in (18).

To derive the optimum solution to $P^*_{j,t}$ from (34) and (35), we make the following assumptions:

Assumption: For $k = 1, 2, 3, ...,$

$$E\left[U_{j,t+k}\right] = U^*_j, \tag{36}$$

$$E\left[Y_{j,t+k}\right] = y^k Y_{j,t}, \tag{37}$$

$$E\left[P_{t+k}\right] = \pi^k P_t, \tag{38}$$

For $k = 0,$

$$E\left[P_t\right] = P_{t-1}, \tag{39}$$

$$E\left[U_{j,t}\right] = U_{j,t-1} \tag{40}$$

These assumptions are all necessary for the problem in (34) - (35) to be tractable while not losing too much rationality. Assumption (36) indicates that the firm expects to invest in the future to adjust the capacity utilization to its optimum. Assumptions (37) and (38) indicate that the expected inflation rate and growth rate of demand are constant over the future. Assumptions (39) and (40) indicate that the firm regards events in the near future as a simple projection from the current observed data.

Proposition 2 *Suppose that assumptions (36) - (40) are satisfied, while $\beta\theta y\pi < 1$. Then, the problem in (34) and (35) allows us to obtain*

$$P_{j,t}^{*} = \alpha_0 P_{t-1} + \alpha_1 P_{t-1} c\left(U_{j,t-1}\right) \tag{41}$$

where

$$\alpha_0 = \frac{\epsilon c_j^{*}\left(\tilde{\beta} - 1\right)}{(\epsilon - 1)\tilde{\beta}}, \quad \alpha_1 = \frac{\epsilon}{(\epsilon - 1)\tilde{\beta}}, \quad \tilde{\beta} = \frac{1}{1 - \beta\theta y\pi} \tag{42}$$

and the function $c(\cdot)$ is the average cost as expressed in (18) with $U_{j,t-1}$ given by (14), and c_j^{*} is the average cost when capacity utilization is at its optimum, as expressed by (22).

The proof of this proposition is given in the appendix.

Given the individual price as expressed in (33) and (41), we now derive the aggregate price P_t. According to Calvo's rule (Calvo, 1983), in period t, the probability that firm j will choose $\tilde{P}_{j,t}$ is θ while the probability that it will choose $P_{j,t}^{*}$ is $1-\theta$. This probability is independent across firms and time. Thus, under the assumption of a representative agent, the dynamics of the aggregate price P_t can be written as

$$P_t = \theta \tilde{P}_{j,t} + (1 - \theta) P_{j,t}^{*} \tag{43}$$

Substituting (33) and (41) into the above while dividing both sides by P_{t-1}, we obtain

$$p_t = (1 - \theta)\alpha_0 + \theta p_{t-1} + (1 - \theta)\alpha_1 c(U_{t-1})$$

Using (42) to express α_0 and α_1, the above equation can further be written as

$$p_t = \alpha_p + \theta p_{t-1} + \alpha_u c(U_{t-1}) \tag{44}$$

where

$$\alpha_p = (1 - \theta)\varepsilon c^{*}\delta\theta, \quad \alpha_u = (1 - \theta)\varepsilon(1 - \delta\theta), \quad \varepsilon = \frac{\epsilon}{\epsilon - 1}, \quad \delta = \beta\pi y$$

5.5 Bounded Rationality: Decision on Production

Once the firm has set the price $P_{j,t}$, it will announce it to the public, and thus the market will generate demand for the firm's output $Y_{j,t}^{d}$. This market demand $Y_{j,t}^{d}$ is delivered to the firm in terms of orders, contracts, etc. As mentioned, the firm's production can simply be expressed as follows: employ labor $L_{j,t}$ according to (13) or (15) to produce $Y^d_{j,t}$ so that $Y_{j,t} = Y^d_{j,t}$. The produced output $Y_{j,t}$ generates $U_{j,t}$, which serves as new information for the firm's decision making in the next period $t+1$.

However, is this way of making decision about production is rational, or can it be derived from an optimization process? We discuss this next.

The firm's production decision problem in period t can be expressed as

$$\max_{Y_{j,t}} P_{j,t}Y_{j,t} - P_t c\left(U_{j,t}\right)Y_{j,t} \tag{45}$$

subject to

$$Y_{j,t} \leq Y^d_{j,t} \tag{46}$$

Again, the function $c(\cdot)$ is the average cost as expressed in (18). Let us first consider the solution without the restriction (46). Define this solution as the optimum solution $Y^*_{j,t}$ as in standard microeconomics. It is not difficult to find that the first-order condition in this case can be written as

$$P_{j,t} = P_t \frac{\omega}{\alpha}\left(\frac{Y^*_{j,t}}{BK_{j,t-1}}\right)^{\frac{1-\alpha}{\alpha}}$$

Solving the above equation for $Y^*_{j,t}$, we obtain

$$Y^*_{j,t} = BK_{j,t-1}\left(\frac{\alpha P_{j,t}}{\omega P_t}\right)^{\frac{\alpha}{1-\alpha}}$$

Therefore, the solution to the problem (45) - (46) can be written as

$$Y_{j,t} = \begin{cases} Y^d_{j,t}, & \text{if } Y^d_{j,t} \leq Y^*_{j,t} \\ Y^*_{j,t}, & \text{otherwise} \end{cases}$$

Figure 5 provides a graphic representation of the determination of $Y_{j,t}$. As the figure shows, if the announced price is given by $P_{j,t}$, the firm's optimum output $Y^*_{j,t}$ is larger than the market demand $Y^d_{j,t}$. In this case, the firm's produced output $Y_{j,t}$ is equal to $Y^d_{j,t}$. If the announced price is equal to $\bar{P}_{j,t}$, the firm's optimum output $\bar{Y}^*_{j,t}$ is less than the corresponding market demand $\bar{Y}^d_{j,t}$. In this case, the firm's produced output $Y_{j,t}$ should be equal to $\bar{Y}^*_{j,t}$.

We have shown that there is a possibility that the firm's production may not satisfy the market demand, but it appears to be at its optimum (or the firm's willingness) when the announced price is equal to $P_{j,t}$. If this occurs often, the economy cannot be regarded as a demand-determined economy and thus the output determination used in this paper does not apply. Is this often the case?

In Figure 6, we find that the marginal revenue curve is always below the demand curve. This makes the price set by the firm $P_{j,t}$ generally higher than the equilibrium price at which the demand curve cuts the marginal cost curve (see point E in the figure). As the marginal cost curve reflects the firm's willingness to supply, we find that it is the monopolistic competition that makes the firm set the price higher than the equilibrium price. This also indicates that generally, the market has an excess supply and therefore the firm's production is often bounded by the market demand.

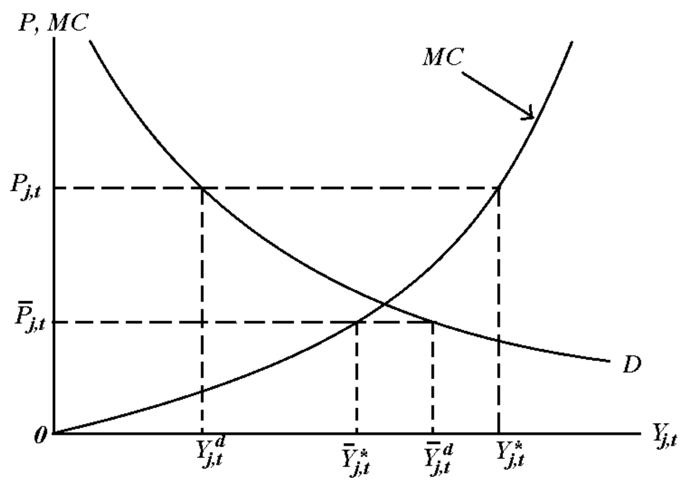

Figure 5: The determination of output, given demand and price

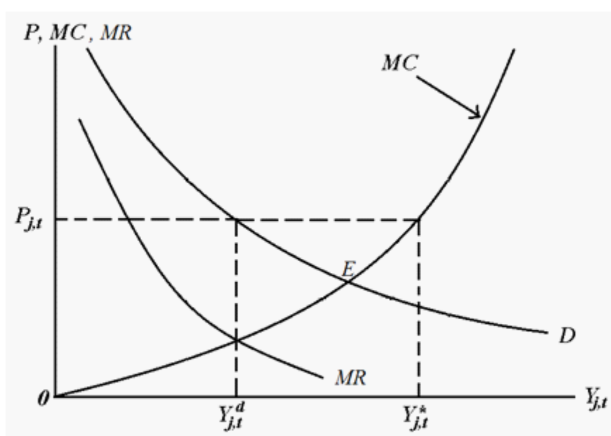

Figure 6: Bounded Rationality under Monopolistic Competition

5.6 The Model as a Standard Dynamic System

The model is now complete; it comprises three behavior equations, the money supply (26), the investment (32), and the pricing (44), with U_t given by (5) and Y_t by (7). From (5) and (7), we obtain

$$\frac{I_t}{K_{t-1}} = sBU_t$$

Substituting this into (32), we obtain

$$U_t = -\frac{\xi_i}{sB} + \frac{\xi_u}{sB}U_{t-1} + \frac{\xi_m}{sB}(m_{t-1} - p_{t-1}) \tag{47}$$

Equations (26), (44), and (47) thus form a standard dynamic system in three dimensions (m_t, p_t, U_t).

6 The Model without Monetary Policy

Suppose $\kappa_p = 0$ and $\kappa_m = 0$. In this case, the system can be transformed into

$$U_t = -\frac{\xi_i}{sB} + \frac{\xi_u}{sB}U_{t-1} + \frac{\xi_m}{sB}(m^* - p_{t-1}) \tag{48}$$

$$p_t = (1-\theta)\varepsilon c^*\delta\theta + \theta p_{t-1} + (1-\theta)(1-\delta\theta)\,\varepsilon c\,(U_{t-1}) \tag{49}$$

This model can be regarded as an economy without a government stabilization policy. We are interested in whether (or under what conditions) the economy can be stabilized.

6.1 The Steady State

The following is the proposition with regard to the steady state of the system (U_t, p_t) composed of (48) and (49).

Proposition 3 *Suppose the cost function $c(U)$ can be linearized as $c'U$ around the steady state. Then, there is a unique steady state $(\bar{U}, \bar{p})$ for the system (U_t, p_t) composed of (48) and (49), which can be expressed as*

$$\bar{U} = \frac{-\xi_i + \xi_m m^* - \xi_m \varepsilon c^* \delta\theta}{sB - \xi_u + \xi_m(1-\delta\theta)\,\varepsilon c'} \tag{50}$$

$$\bar{p} = \varepsilon c^*\delta\theta + (1-\delta\theta)\,\varepsilon c'\bar{U} \tag{51}$$

The proof of this proposition is given in the appendix. The assumption of a linearized cost function around the steady state greatly simplifies our analysis. Without losing generality, it allows us to avoid the irreversible problem when using the cost function (18) to derive the steady state.

6.2 The Stability

Next, we analyze the stability pertaining to the system (U_t, p_t). The Jacobian matrix of the system (U_t, p_t) can be written as

$$J = \begin{bmatrix} \frac{\xi_u}{sB} & -\frac{\xi_m}{sA} \\ (1-\theta)(1-\delta\theta)\varepsilon c' & \theta \end{bmatrix}$$

where c' is the derivative of $c(U)$ evaluated at the steady state $\bar{U}$. Thus, the characteristic equation takes the form of

$$\lambda^2 - \left(\frac{\xi_u}{sB} + \theta\right)\lambda + \frac{\xi_u}{sB}\theta + \frac{\xi_m}{sB}(1-\theta)(1-\delta\theta)\varepsilon c' = 0$$

Solving the above equation for λ's, we obtain two eigenvalues, which can be expressed as

$$\lambda_{1,2} = \frac{1}{2}\left\{\frac{\xi_u}{sB} + \theta \pm \sqrt{\left(\frac{\xi_u}{sB} + \theta\right)^2 - 4\left[\frac{\xi_u}{sB}\theta + \frac{\xi_m}{sB}(1-\theta)(1-\delta\theta)\varepsilon c'\right]}\right\}$$

The following is the proposition regarding the properties of our two eigenvalues $\lambda_{1,2}$.

Proposition 4 *Assume $\lambda_1 \geq \lambda_2$.*

1. *Suppose that the condition*

$$\left(\frac{\xi_u}{sB} - \theta\right)^2 \geq 4\frac{\xi_m}{sB}(1-\theta)(1-\delta\theta)\varepsilon c' \tag{52}$$

   holds. In this case, the two eigenvalues λ_1 and λ_2 are both real. Furthermore, if

$$1 - \frac{\xi_u}{sB} + \frac{\xi_m}{sB}\varepsilon c'(1-\delta\theta) > 0 \tag{53}$$

   is satisfied, then

$$1 > \lambda_1 \geq \lambda_2 > -1$$

2. *Suppose that the condition (52) cannot hold so that the two eigenvalues λ_1 and λ_2 are complex conjugate. The modulus of the two eigenvalues denoted as $|\lambda_{1,2}|$ can be either below or above 1 depending on the castellation of the structure parameters. In particular, if $(2-\theta)sB - \xi_u > 0$, then there exists a critical value of ξ_m denoted as ξ_m^* with*

$$\xi_m^* = \frac{sB - \xi_u\theta}{\varepsilon c'(1-\theta)(1-\delta\theta)} \tag{54}$$

   such that in the neighborhood of ξ_m^*:

   (a) $|\lambda_{1,2}| < 1$ when $\xi_m < \xi_m^*$;
   (b) $|\lambda_{1,2}| = 1$ when $\xi_m = \xi_m^*$;
   (c) $|\lambda_{1,2}| > 1$ when $\xi_m > \xi_m^*$.

The proof of this proposition is given in the appendix.

6.3 The Strength of the Stabilization Mechanism

Before providing the stability analysis with the proposition, we shall first observe the stabilizing and destabilizing mechanisms entailed in the model. The stabilizing mechanism is the price adjustment. According to equation (49), greater utilization of capacity U_{t-1} leads to a higher inflation rate p_t. Given a stable money supply m^*, the financial resources for investment (in real terms) are reduced, which leads to a decrease in both the investment rate (see equation (32)) and capacity utilization (see equation (48)). The destabilization mechanism still arises from the investment adjustment and is reflected as $\frac{\xi_u}{sB} > 1$ (see equation (48)).

Given the other structural parameters, such as ξ_u, s, and B among others, we find that whether the stabilization mechanism is strong enough to overcome the destabilization mechanism depends on

- θ, which reflects the stickiness of price adjustment: the larger the value of θ, the more sticky the price adjustment; and
- ξ_m, which reflects the effect that price adjustment has on investment: the larger the value of ξ_m, the greater the effect of the price adjustment on investment.

6.4 Stability when ξ_m is not Large

We now discuss the stability of our model (U_t, p_t) by relying on Proposition 4. Note that condition (52) can also be written as

$$\xi_m \leq \frac{\left(\frac{\xi_u}{sB} - \theta\right)^2 sB}{4(1-\theta)(1-\delta\theta)\,\varepsilon c'}$$

Thus, we can regard (52) as the condition in which ξ_m is not large enough. According to Proposition 4, the two eigenvalues λ_1 and λ_2 are both real in this case, so there is no cyclical behavior in the economy. The trajectories of U_t and p_t can either be monotonically divergent or monotonically convergent. Whether convergent or divergent depends on whether (53) is satisfied. Without difficulty, one may find that condition (53) can also be written as

$$\theta < \frac{sB - \xi_u + \xi_m \varepsilon c'}{\xi_m \varepsilon c' \delta}$$

Setting

$$\theta^* \equiv \frac{sB - \xi_u + \xi_m \varepsilon c'}{\xi_m \varepsilon c' \delta} \tag{55}$$

provided that $\theta^* \in (0, 1)$, we find that a small perpetuation from θ^* will change the stability of (U_t, p_t). Specifically, if $\theta < \theta^*$, or the price is not too sticky, the economy can monotonically (rather than cyclically) converge to the steady state

$(\bar{U}, \bar{p})$; however, if $\theta > \theta^*$, or the price is too sticky, the system is monotonically divergent, as in the Harrodian economy. Therefore, a pitchfork bifurcation exists around θ^*.

Figure 7 and 8 provide the numerical simulations of these two situations.

6.5 Numerical Simulation

The parameters used for our simulations are given in Table 1.

Table 1 benchmark Parameters

ξ_u	ξ_m	B	m^*	s	δ	ε	c'	c^*	ξ_i	θ
0.220	0.040	0.655	1.130	0.300	1.000	1.200	0.962	0.962	0.022	0.55

The parameters ξ_u, ξ_m, θ, and B are taken from Gong and Gao (2013), where the similar investment and price functions, as in this paper, are estimated using annual data from China. The parameter ε is taken from Christiano et al. (2005). The parameter $\bar{m}$ can be regarded as the average gross rate of growth of the money supply and is therefore set within a reasonable range given the data on the Chinese economy. The parameters s and δ are also set within their reasonable ranges. We assume that the parameters c' and c^* are initially equal, which indicates that in the steady state $\bar{U}$, the marginal cost curve cuts the average cost curve at this minimum (see Figure 4). The values of c', c^*, and ξ_i are then computed based on the assumption that the steady states $\bar{U}$ and $\bar{p}$ are equal to 0.8 and 1.05, respectively. In particular, given $\bar{U} = 0.8$, $\bar{p} = 1.05$ and the other benchmark parameters, we compute c', c^*, and ξ_i as follows:

$$c^* = c' = \frac{\bar{p}}{\varepsilon \left[\delta\theta + (1 - \delta\theta)\,\bar{U}\right]} \tag{56}$$

$$\xi_i = \xi_m(m^* - \bar{p}) - (sB - \xi_u)\bar{U} \tag{57}$$

Thus, the benchmark case, for which the parameters are given in Table 1, is the case that most closely matches the actual Chinese economy. Given these benchmark parameters, we find that the bifurcation value θ^*, as expressed in (55), is equal to 0.48985556. Therefore, the benchmark parameter θ in Table 1 is larger than θ^*.

Figure 7 simulates the model economy using the benchmark parameters with the initial condition set at 95% of the corresponding steady states. As the figure shows, when ξ_m is not large and $\theta > \theta^*$, that is, price adjustment does not have a significant effect on investment while pricing is too sticky, the economy is divergent, as in the Harrodian economy.

In contrast, in the case of $\theta < \theta^*$, or if the price adjustment is somehow more flexible, the economy can be stabilized through monotonic convergence to the steady state, as shown in Figure 8.

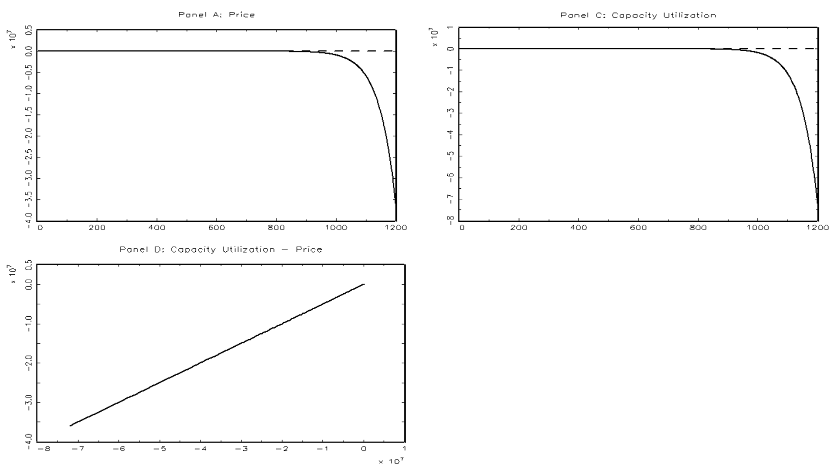

Figure 7: The dynamics of (p_t, U_t) in the benchmark case ($\theta > \theta^*$)

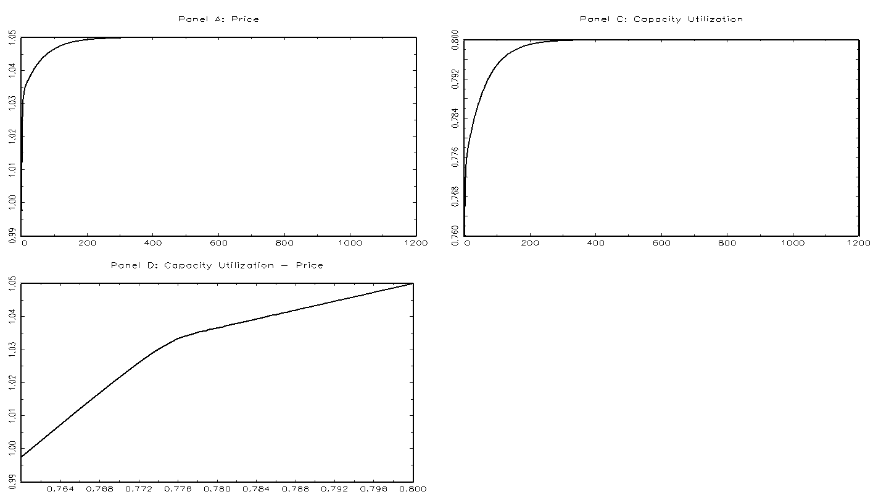

Figure 8: The dynamics of (p_t, U_t) in the case of real eigenvalues, and $\theta < \theta^*$ ($\theta = 0.45$)

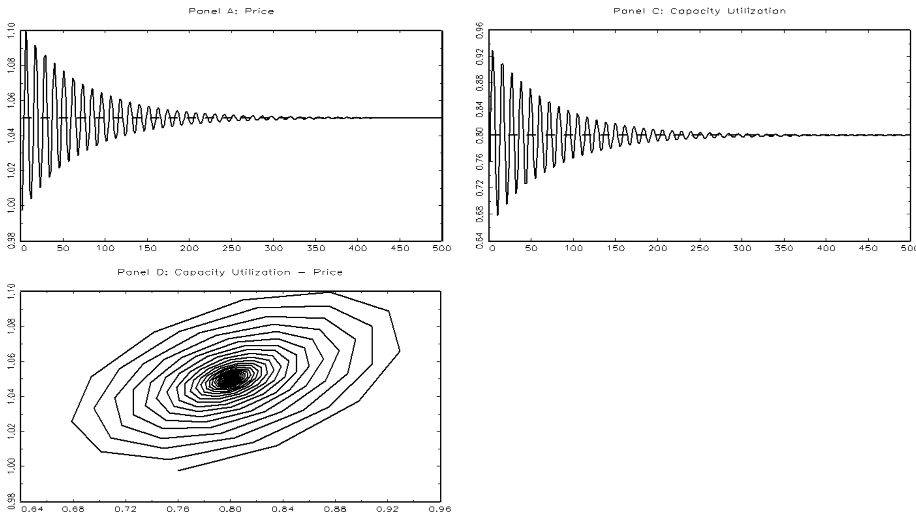

Figure 9: The dynamics of (p_t, U_t), in the case of complex eigenvalues and $\xi_m < \xi_m^*$ ($\xi_m = 0.3$)

6.6 Stability when ξ_m is Large

Next, we consider the situation where condition (52) is not satisfied. For this, we allow ξ_m be increase until condition (52) is no longer satisfied. As expressed in Proposition 4, in this case, the two eigenvalues λ_1 and λ_2 are complex conjugate, indicating that cyclical behavior is expected in the economy. However, the cycle can be either convergent or explosive depending on the increase in ξ_m.

Suppose that parameter ξ_m increases from 0.04 to 0.3 while the other parameters remain at their benchmark level, as illustrated in Figure 9. In this case, the economy cyclically converge to the steady state.

However, when ξ_m increases too much, for instance to 0.35, the economy will become cyclically explosive as shown Figure 10. This indicates that a Hopf-bifurcation exists between 0.3 and 0.35 with respect to parameter ξ_m. Indeed, given the other benchmark parameters in Table 1, the Hopf-bifurcation ξ_m^* can be computed by (54), from which we find that $\xi_m^* = 0.32253297$.

We thus find that in the absence of a government stabilization policy, the stability of the economy depends on two parameters, θ and ξ_m (given the others), that reflect the strength of the price adjustment of the stabilization mechanism. Specifically, if ξ_m (which reflects the effectiveness of price adjustment on investment) is not large enough to generate cyclical behavior in the economy, the

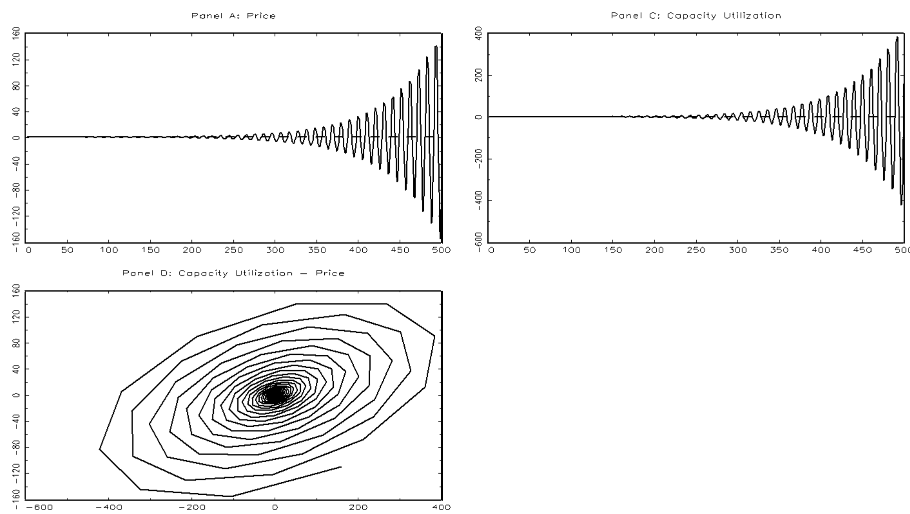

Figure 10: The dynamics of (p_t, U_t), in the case of complex eigenvalues and $\xi_m > \xi_m^*$ $(\xi_m = 0.35)$

economy could be monotonically divergent as in the Harrodian economy. This possibility becomes large when θ (which reflects the stickiness of pricing) tends to be larger, or the price becomes more stickier. Another possible case of instability occurs when ξ_m is too large. In this case, the economy is cyclically explosive. Empirically, at least according to the data from China, the first type of instability is more likely to occur.

7 The Model with Monetary Policy

In this section, we introduce a monetary policy into the model. Our purpose is to exam whether the economy can be stabilized if the model initially is monotonically divergent.

7.1 The Steady State

After introducing the monetary policy, the system is composed of (26), (47), and (49). The proposition with regard to the steady state of our new system (m_t, U_t, p_t) is given as follows.

Proposition 5 *Suppose that the cost function $c(U)$ can be linearized as $c'U$ around the steady state. Then, there is a unique steady state $(\bar{m}, \bar{U}, \bar{p})$ for the* *system* (m_t, U_t, p_t) *composed of (26), (47) and (49), which can be expressed as*

$$\begin{bmatrix} \bar{m} \\ \bar{U} \\ \bar{p} \end{bmatrix} = \begin{pmatrix} 1 & 0 & \frac{\kappa_p}{\kappa_m} \\ -\frac{\xi_m}{sB-\xi_u} & 1 & \frac{\xi_m}{sA-\xi_u} \\ 0 & -(1-\delta\theta)\,\varepsilon c' & 1 \end{pmatrix}^{-1} \begin{bmatrix} \frac{\kappa_p}{\kappa_m} p^* + m^* \\ \frac{-\xi_i}{sB-\xi_u} \\ \varepsilon c^* \delta\theta \end{bmatrix}$$

The proof of this proposition is trivial and thus is not provided here.

7.2 Stability

Next, we examine the stability of the system (m_t, U_t, p_t). The proposition is as follows.

Proposition 6 *Let*

$$\begin{aligned} \sigma &= \frac{\xi_m \varepsilon c' (1-\theta)(1-\delta\theta)}{sB} \\ a &= \left(1 - \frac{\xi_u \theta}{sB} - \sigma\right)\left[1 + (1-\kappa_m)^2 \left(\frac{\xi_u \theta}{sB} + \sigma\right) - (1-\kappa_m)\left(\theta + \frac{\xi_u}{sB}\right)\right] \\ b &= \left[(1-\kappa_m)\left(2\frac{\xi_u \theta}{sB} + 2\sigma_2 - 1\right) - \theta + \frac{\xi_u}{sB}\right] \end{aligned}$$

If the following conditions hold,

$$(2-\kappa_m)\left[(1+\theta)\left(1+\frac{\xi_u}{sA}\right)+\sigma\right] > \kappa_p \sigma > -\kappa_m\left[\left(1-\frac{\xi_u}{sA}\right)(1-\theta)+\sigma\right] \quad (58)$$

$$a + b\sigma\kappa_p - (\sigma\kappa_p)^2 > 0 \quad (59)$$

then the system (m_t, U_t, p_t) *is asymptotically stable at the steady state* $(\bar{m}, \bar{U}, \bar{p})$. *Furthermore, if (58) is satisfied, the system undergoes a Hopf-bifurcation at* $a + b\sigma\kappa - (\sigma\kappa)^2 = 0$.

The proof of this proposition is given in the appendix.

The economic meaning of this proposition can be expressed as follows. First, we find that $\left(1 - \frac{\xi_u}{sB}\right)(1-\theta)+\sigma$ on the right side of condition (58) is negative if condition (53) in proposition 4 is not satisfied or if the economy is monotonically divergent without a monetary policy. Therefore, condition (58) indicates that if the economy starts off monotonically divergent without a monetary policy, then the monetary policy should be exerted (i.e., $\kappa_p > 0$) for the economy to be stabilized. However, the effect of the monetary policy (represented by κ_p given κ_m) should be in appropriate: neither absent, i.e., $\kappa_p > 0$, nor over-exerted, i.e.,

$$\kappa_p < (2-\kappa_m)\left[(1+\theta)\left(1+\frac{\xi_u}{sB}\right)+\sigma\right]/\sigma$$

Next, we examine condition (59). Denote the left side of condition (59) as $f(\kappa_p)$, that is, $f(\kappa_p) \equiv a + b\sigma\kappa_p - (\sigma\kappa_p)^2$. Apparently, f is quadratic in κ_p. This

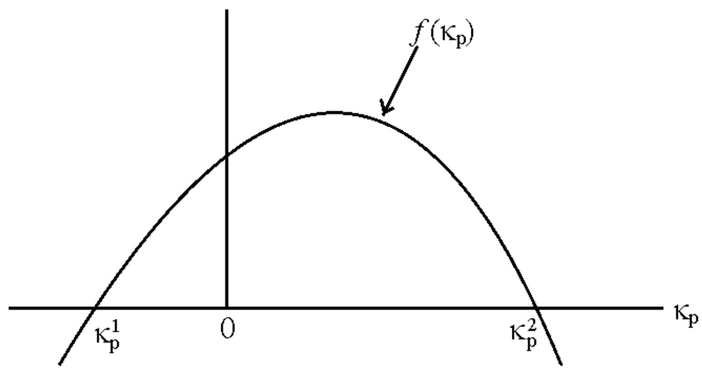

Figure 11: The quaratic function of $f(\kappa_p)$: the identification of birfurcations

indicates that there are two possible roots for $f(\kappa_p) = 0$, and hence two possible bifurcations. Without losing generality, we denote these two bifurcations as κ_p^1 and κ_p^2, with $\kappa_p^1 < \kappa_p^2$. As $f''(\kappa_p) < 0$, the possible shape of $f(\kappa_p)$ is shown in Figure 11. Thus, if $\kappa_p > \kappa_p^2$ or $\kappa_p < \kappa_p^1$, the economy cannot be stabilized. The economy can only be stabilized at the steady state if $\kappa_p^1 < \kappa_p < \kappa_p^2$ so that $f(\kappa_p) < 0$ (provided that $\kappa_p > 0$).

7.3 Numerical Simulation

The same parameters (Table 1) are again used for our numerical simulation with c', c^*, and ξ_i still computed by (56) and (57), respectively. The exception is that $\bar{p}$ is now replaced by p^*, which we set to 1.05, and $\bar{U}$ is still given by 0.8. We find that in this case, $\bar{p} = 1.05$ and $\bar{m} = m^* = 1.13$. We also know that in this case, if there is no monetary policy (or $\kappa_p = \kappa_m = 0$), the economy is monotonically divergent, as shown in Figure 7. We set $\kappa_m = 0.433$, which is the estimated parameter by Gong and Lin (2008).

Given these parameters, we compute the two bifurcations κ_p^1 and κ_p^2 according to $f(\kappa_p) = 0$. We find that κ_p^1 is negative, equal to -32.143 and κ_p^2 is positive, equal to 1.207 (see Figure 11). Therefore, we only need to exam how the perpetuation of κ_p from κ_p^2 can change the stability of the economy.

Figure 12 provides the simulation when $\kappa_p < \kappa_p^2$. As the figure shows, and as we expected in this case, the economy can be stabilized. Figure 13 simulates the dynamics of (m_t, U_t, p_t) when $\kappa_p > \kappa_p^2$. In this case, the economy is cyclically explosive, indicating that the effort of monetary policy is too strong.

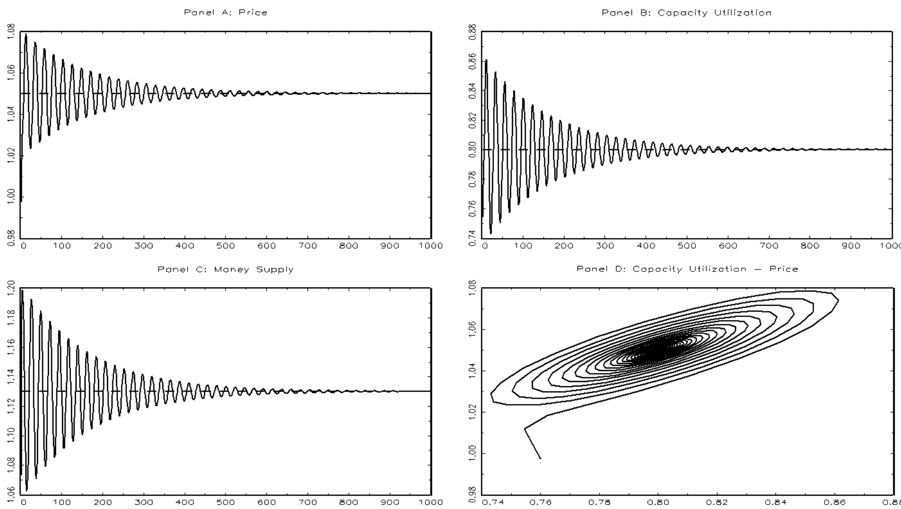

Figure 12: The dynamics of (m_t, p_t, U_t), the case of $\kappa_p < \kappa_p^1$ $(\kappa_p = 1.13)$

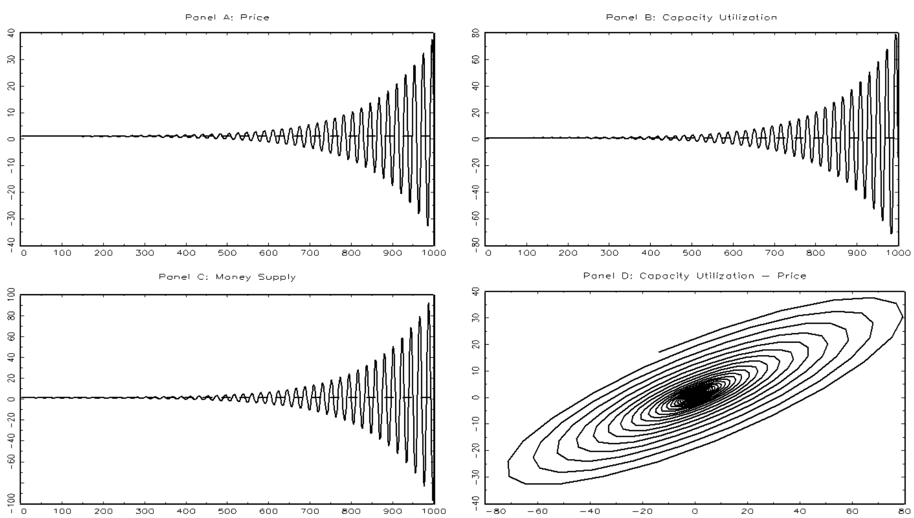

Figure 13: The dynamics of (m_t, p_t, U_t), the case of $\kappa_p > \kappa_p^2$ $(\kappa_p = 1.3)$

8 Discussion and Conclusion

We aim to answer two main questions in this paper. First, is there a destabilization mechanism in a market economy? Second, if such as mechanism exists, how strong is its destabilization force, and is it possible for it to be stronger than the stabilizing force of price adjustment?

Regarding to the first question, we find that a mechanism does exist that destabilizes the economy. We demonstrate an important example: the investment adjustment underlying Harrod's instability (or knife-edge) puzzle (Harrod, 1939). We construct a dynamic optimization model from which the investment function is derived. On the basis of this investment function, we prove that a destabilization mechanism in a simple economy (such as Harrodian economy) exists due to the adjustment in investment.

With respect to the second question, we first find that the price adjustment is differ between neoclassical and Keynesian economics. Neoclassical economics assumes a competitive economy, in which case, stabilization by price adjustment is very efficient and the economy can always be stabilized. However, Keynesian economics assumes a non-competitive economy in which price adjustments may be less frequency, but sticky. This indicates that it is quite possible that a price adjustment may not exert a sufficiently strong force to overcome the destabilizing force. Therefore, the economy could become unstable without any further stabilization mechanism, such as from government intervention. We also identify the critical conditions (reflected by the stickiness θ and the effectiveness ξ_m of price adjustment) under which the economy becomes unstable without a government stabilization policy.

Finally, we should point out that our discussion of stabilization and destabilization mechanisms may not offer a complete explanation, because other similar mechanisms may exist. Therefore, rather than aiming to answer the question of whether government stabilization policy is necessary, we merely demonstrate that business cycles can be understood as an interaction between stabilizing and destabilizing mechanisms, yet the latter are often ignored in the economics literature.

9 Appendix

9.1 Proof of Proposition 1

Expressing $I_{j,t+k}$ in (20) in terms of (21) and $c_{j,t+k}$ in terms of (18), we find that the problem (20) can be re-written as

$$\max E \sum_{k=0}^{\infty} \beta^k \left\{ P_{j,t+k} Y_{j,t+k} - P_{t+k} c_{j,t+k}(U_{j,t+k}) Y_{j,t+k} - (1+r) P_{t+k} \left[K_{j,t+k} - (1-d_j) K_{j,t+k-1} \right] \right\}$$

Note that from (14) we find that $U_{j,t+k}$ is also a function of $K_{j,t+k-1}$. In particular,

$$\frac{\partial U_{j,t+k}}{\partial K_{j,t+k-1}} = -\frac{Y_{j,t+k}}{B\left(K_{j,t+k-1}\right)^2} = -\frac{B\left(U_{j,t+k}\right)^2}{Y_{j,t+k}}$$

Therefore, the problem becomes the choice of the sequence $\{K_{j,t+k}\}_{k=0}^{\infty}$. The Euler equation for this problem can be written as

$$E\beta^k P_{t+k}\left[\frac{(1-\alpha)\omega}{\alpha}\left(U_{j,t+k}\right)^{\frac{1-\alpha}{\alpha}-1} B\left(U_{j,t+k}\right)^2 - v_j\left(U_{j,t+k}\right)^{-2}\left(U_{j,t+k}\right)^2 + (1+r)(1-d_j)\right] - P_{t+k-1}(1+r)\beta^{k-1} = 0$$

which can further be simplified as

$$E\beta\pi\left[\frac{(1-\alpha)\omega}{\alpha}\left(U_{j,t+k}\right)^{\frac{1}{\alpha}} B - v_j + (1+r)(1+d_j)\right] = 1+r$$

This equation allows us to obtain

$$\left(U_{j,t+k}\right)^{\frac{1}{\alpha}} = \frac{(1+r)/(\beta\pi) - (1+r)(1-d_j) + v_j}{\frac{1-\alpha}{\alpha}B\omega}$$

We therefore prove the proposition.

9.2 Proof of Proposition 2

Substituting (35) into (34), we obtain

$$\max_{P_{j,t}^*} E\sum_{k=0}^{\infty}\beta^k\theta^k\left[\pi^k P_{j,t}^*\left(\frac{P_{t+k}}{\pi^k P_{j,t}^*}\right)^{\epsilon} Y_{t+k} - P_{t+k}c(U_{j,t+k})\left(\frac{P_{t+k}}{\pi^k P_{j,t}^*}\right)^{\epsilon} Y_{t+k}\right]$$

The first-order condition can thus be expressed as

$$E\sum_{k=0}^{\infty}\beta^k\theta^k\left[\pi^k\left(\frac{P_{t+k}}{\pi^k P_{j,t}^*}\right)^{\epsilon} Y_{t+k} - \epsilon\pi^k P_{j,t}^*\left(\frac{P_{t+k}}{\pi^k P_{j,t}^*}\right)^{\epsilon-1} Y_{t+k}\frac{P_{t+k}}{\pi^k\left(P_{j,t}^*\right)^2}\right] +$$

$$E\sum_{k=0}^{\infty}\beta^k\theta^k\left[\epsilon P_{t+k}c(U_{j,t+k})\left(\frac{P_{t+k}}{\pi^k P_{j,t}^*}\right)^{\epsilon-1} Y_{t+k}\frac{P_{t+k}}{\pi^k\left(P_{j,t}^*\right)^2}\right] = 0$$

which can be simplified as

$$E\sum_{k=0}^{\infty}\beta^k\theta^k\left[\pi^k(1-\epsilon)Y_{j,t+k} + \epsilon P_{t+k}c(U_{j,t+k})\frac{Y_{j,t+k}}{P_{j,t}^*}\right] = 0$$

Solving the above equation for $P_{j,t}^*$, we obtain

$$P_{j,t}^* = \frac{\epsilon E\sum_{k=0}^{\infty}\beta^k\theta^k\left[P_{t+k}c(U_{j,t+k})Y_{j,t+k}\right]}{(\epsilon-1)E\sum_{k=0}^{\infty}\beta^k\theta^k\left[\pi^k Y_{j,t+k}\right]}$$

Using assumptions (**??**) to (40), the above equation can further be written as

$$\begin{aligned} P_{j,t}^{*} &= \frac{\epsilon E \sum_{k=1}^{\infty} \beta^k \theta^k \pi^k y^k \left[P_t c(U_j^*) Y_{j,t} \right] + \epsilon E \left[P_t c(U_t) Y_{j,t} \right]}{(\epsilon - 1) E \sum_{k=0}^{\infty} \beta^k \theta^k \pi^k y^k \left[Y_{j,t} \right]} \\ &= \frac{\epsilon E \sum_{k=0}^{\infty} \beta^k \theta^k \pi^k y^k \left[P_t c(U_j^*) Y_{j,t} \right] + \epsilon E \left[P_t c(U_t) Y_{j,t} \right] - \epsilon \left[P_t c(U_j^*) Y_{j,t} \right]}{(\epsilon - 1) E \sum_{k=0}^{\infty} \beta^k \theta^k \pi^k y^k \left[Y_{j,t} \right]} \\ &= \frac{\epsilon \tilde{\beta} P_{t-1} c(U_j^*) + \epsilon P_{t-1} c(U_{t-1}) - \epsilon P_{t-1} c(U_j^*)}{(\epsilon - 1) \tilde{\beta}} \end{aligned}$$

Simplifying the above, we obtain (41) as in the proposition.

9.3 Proof of Proposition 3

Let $p_t = p_{t-1} = \bar{p}$ and $U_t = U_{t-1} = \bar{U}$. Equation (49) allows us to obtain

$$\bar{p} = \varepsilon c^* \delta\theta + (1 - \delta\theta)\, \varepsilon c' \bar{U} \tag{60}$$

This is the same as equation (51) in the proposition. Meanwhile, from (48),

$$sB\bar{U} = -\xi_i + \xi_u \bar{U} + \xi_m \bar{m} - \xi_m \bar{p} \tag{61}$$

Substituting (60) into (61), we obtain

$$sB\bar{U} = -\xi_i + \xi_u \bar{U} + \xi_m \bar{m} - \xi_m \left[\varepsilon c^* \delta\theta + (1 - \delta\theta)\, \varepsilon c' \bar{U} \right]$$

Re-organizing the above equation, we obtain (50) as in the proposition.

9.4 Proof of Proposition 4

First, we prove section 1 of the proposition. For notational convenience, we denote

$$a \equiv \frac{\xi_u}{sB}\theta + \frac{\xi_m}{sB}(1-\theta)\varepsilon\,(1 - \delta\theta)\, c', \quad a > 0 \tag{62}$$

Thus, for λ_1 and λ_2 to be real, we require that

$$\left(\frac{\xi_u}{sB} + \theta_p \right)^2 - 4a > 0$$

Re-organizing the above equation, we obtain condition (52) as in the proposition.

Suppose now that

$$1 > \lambda_1 \geq \lambda_2 > -1$$

is satisfied. Then for $1 > \lambda_1$, we should have

$$\frac{1}{2} \left[\frac{\xi_u}{sB} + \theta + \sqrt{\left(\frac{\xi_u}{sB} + \theta \right)^2 - 4a} \right] < 1$$

which can further be written as

$$\sqrt{\left(\frac{\xi_u}{sB}+\theta\right)^2-4a}<2-\frac{\xi_u}{sB}-\theta \tag{63}$$

Next, for $\lambda_2>-1$, we have

$$\frac{1}{2}\left[\frac{\xi_u}{sB}+\theta-\sqrt{\left(\frac{\xi_u}{sB}+\theta\right)^2-4a}\right]>-1$$

which can further be written as

$$\frac{\xi_u}{sB}+\theta_p+2>\sqrt{\left(\frac{\xi_u}{sB}+\theta_p\right)^2-4a} \tag{64}$$

Comparing (63), we find that if condition (63) is satisfied, condition (64) must be satisfied. Thus, we only need to continue with condition (63).

Taking the square of both sides of (63), we obtain

$$\left(\frac{\xi_u}{sB}+\theta\right)^2-4a<\left(2-\frac{\xi_u}{sB}-\theta\right)^2$$

Re-organizing the above:

$$\left(\frac{\xi_u}{sB}+\theta\right)^2-\left(2-\frac{\xi_u}{sB}-\theta\right)^2<4a$$

which can further be simplified as

$$\frac{\xi_u}{sB}+\theta-1<a$$

Expressing a in terms of (62), the above equation can be re-written as

$$0<(1-\theta)\left(1-\frac{\xi_u}{sB}\right)+(1-\theta)\frac{\xi_m}{sB}\varepsilon c'(1-\delta\theta)$$

which is indeed condition (53) as in the proposition. We thus prove section 1 of the proposition.

Next, suppose condition (52) is not satisfied. Assume that $|\lambda_{1,2}|$, which is indeed a as expressed in (62), is equal to 1, that is,

$$\frac{\xi_u}{sB}\theta+\frac{\xi_m}{sB}(1-\theta)(1-\delta\theta)\varepsilon c'=1$$

Solving the above equation for ξ_m, we obtain

$$\xi_m^*=\frac{sB-\xi_u\theta}{\varepsilon c'(1-\theta)(1-\delta\theta)} \tag{65}$$

Given this ξ_m^*, we now prove that the eigenvalue $\lambda_{1,2}$ are complex conjugate. This requires that $a_1^2 - 4a_2 < 0$, that is,

$$\left(\frac{\xi_u}{sB} + \theta\right)^2 - 4\left[\frac{\xi_u}{sB}\theta + \frac{\xi_m^* \varepsilon c'}{sB}(1-\theta)(1-\delta\theta)\right] < 0 \tag{66}$$

Expressing ξ_m^* in (66) in terms of (65), we obtain

$$\frac{\xi_u}{sB} + \theta < 2$$

This is the condition $(2-\theta)\, sB - \xi_u > 0$ as in the proposition. We therefore prove section 2 of the proposition.

9.5 Proof of Proposition 6

The Jacobian matrix of system (m_t, U_t, p_t) can be written as

$$J = \begin{bmatrix} 1-\kappa_m & 0 & -\kappa_p \\ \frac{\xi_m}{sB} & \frac{\xi_u}{sB} & -\frac{\xi_m}{sB} \\ 0 & (1-\theta)(1-\delta\theta)\,\varepsilon c^{'} & \theta \end{bmatrix}$$

The characteristic function $\det|\lambda I - J| = 0$ takes the form of

$$\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0$$

where

$$\begin{aligned} c_1 &= -\left(1-\kappa_m + \frac{\xi_u}{sB} + \theta\right) \\ c_2 &= \frac{\xi_u}{sB}\theta + \frac{\xi_m}{sB}(1-\theta)(1-\delta\theta)\,\varepsilon c^{'} + (1-\kappa_m)\left(\theta + \frac{\xi_u}{sB}\right) \\ c_3 &= -(1-\kappa_m)\frac{\xi_u}{sB}\theta - (1-\kappa_m-\kappa_p)\frac{\xi_m}{sB}(1-\theta)(1-\delta\theta)\,\varepsilon c^{'} \end{aligned}$$

For the convenience of our proof, we first establish the following theorem provided by Elaydi (1996) and Sonis (2000):

Theorem 7 *Let*

$$\begin{aligned} \pi_1 &= 1 + c_1 + c_2 + c_3 \\ \pi_2 &= 1 - c_1 + c_2 - c_3 \\ \pi_3 &= 1 - c_2 + c_1c_3 - c_3^2 \end{aligned}$$

If $\pi_i (i = 1, 2, 3) > 0$, and $c_3 < 3$, then the module of eigenvalues of J is less than 1. Meanwhile, the system undergoes a bifurcation at $\pi_3 = 0$.

Applying the above theorem, we find that

$$\begin{aligned}
\pi_1 &= \kappa_m \left(1 - \frac{\xi_u}{sB}\right)(1-\theta) + (\kappa_p + \kappa_m)\sigma \\
\pi_2 &= (2-\kappa_m)(1+\theta)\left(1 + \frac{\xi_u}{sB}\right) + (2 - \kappa_m - \kappa_p)\sigma \\
\pi_3 &= a + b\sigma\kappa_p - (\sigma\kappa_p)^2
\end{aligned}$$

Thus, $\pi_1 > 0 \Leftrightarrow \kappa_p\sigma > -\kappa_m[(1-\frac{\xi_u}{sB})(1-\theta)+\sigma]$, and $\pi_2 > 0 \Leftrightarrow (2-\kappa_m)\,[(1+\theta)(1+\frac{\xi_u}{sB})+\sigma] > \kappa_p\sigma$. Putting $\pi_1 > 0$ and $\pi_2 > 0$ together, we obtain condition (58) as in the proposition. $\pi_3 > 0 \Leftrightarrow a + b\sigma\kappa_p - (\sigma\kappa_p)^2 > 0$, which is condition, (59) as in the proposition.

We thus complete the proof of the proposition.

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