Financial Markets, Information Acceleration, and Resource Misallocation

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Motivation

- a small shock on the financial sector can have a large impact on the aggregate economy: evident by the recent financial crisis
- two potential channels of financial accelerator
  1. financing channel (leverage, beta multiplier): A large literature on the financial accelerator emphasizes on the lending (credit) channel (B&G (1989), K&M (1997)
  2. information channel: this paper emphasizes informational accelerator in an extended Grossman-Stiglitz (1980) model with real investment and macroeconomic fluctuations.

The information channel

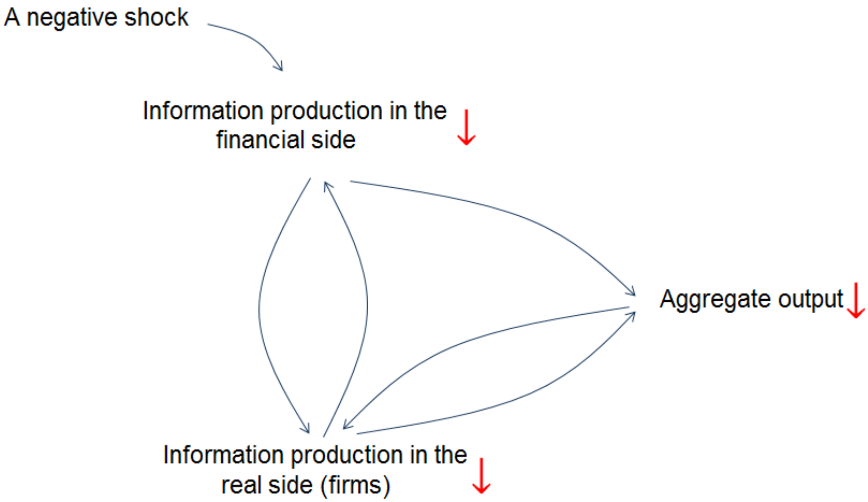
- Mutual learning between firms and financial markets, built on two previous literature
  1. financial markets learn from firms' disclosure (a large accounting literature)
  2. firm mangers learn from financial prices (Bond et al. (2012)) to make better investment decision
- an example: oil production companies look at the oil futures to decide its production and financial market looks at the financial reports from these companies to trade oil futures.

The information channel

Financial market helps efficiently allocate resources by producing right information

- resources may go to wrong projects due to little information about who are good and who are not
- an alternative micro-foundation for resource misallocations (Hsieh and Klenow (2008))
    - quantitatively important as documented by David, Hoppenhayn and Venkateswaran (2015)
    - the conventional view that resource misallocation is mainly due to the financial channel is challenged by Midrigan and Xu (2015): financial constraints lead to little misallocation.

Informational accelerator



Main Results

- A small shock on the financial sector that impairs its ability to perform price discovery can have a large impact on the aggregate economy
- In general equilibrium, aggregate real output and financial market efficiency feedback and reinforce each other
- The aggregate TFP, which maps the degree of misallocation of resources, and the aggregate investment both are decreasing in information precision.
- The information acquisition yields possible self-fulfilling crisis

The Road Map

1. a partial equilibrium model with exogenous informaiton to understand
   - financial market equilibrium
   - the firm investment decision
2. endogenous information to understand
   - information acquisition of the firm
   - information acquisition of financial market
   - their interactions.
3. a general equilibrium macro model to understand
   - how macroeconomic condition interact with 1 and 2.
   - resource misallocation

Model Setup: A Partial Equilibrium with Exogenous Information

We first consider an model with exogenous Information, the model includes

1. informed investor with measure of λ and uninformed investors with measure of $1-\lambda$
2. a monopoly

Investors trade a risky asset, a derivitative. Its final payoff is indexed to the total revenue of the monopoly. The final payoff of monopoly is affected by two type of uncertainties

1. demand uncertainty, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, known to informed investors
2. supply uncertainty, $a \sim \mathcal{N}(0, \sigma_a^2)$, known to the monopoly with noisy and disclosed to all investors: $s = a + e$, where $e \sim \mathcal{N}(0, \sigma_e^2)$
3. the total supply of derivitative is $x \sim \mathcal{N}(0, \sigma_x^2)$

The timing of events

1. In the first period, investors trade a risky asset (derivitative) at q and a risk free asset with gross return normalized to be unity
2. the monopoly firm decide its investment K
3. firm's sale revenue realizes and all uncertainties resolve in period 2. Investors obtain $v = \log(P \times Y)$ for each unit of risk asset

The firm's investment problem

- Demand uncertainty:

$$Y = (\frac{1}{P})^{\theta} C \exp(\frac{1}{\theta}\varepsilon), \tag{1}$$

where $\theta > 1$ is the price elasticity, C standards for aggregate demand (to be endogenized in general equilibrium), and ε is a firm specific demand shock.

- Supply uncertainty: The monopoly production Y depends on its investment and a technology shock

$$Y = \exp(a) K \tag{2}$$

where K is the firm's investment.

The firm's investment problem

- The firm needs to decide its investment based on its signal $s = a + e$ and market price for the risk asset, q. It learns ε from asset price q.
- The investment problem is hence to solve

$$\max_K \mathbb{E}\left\{\left[\exp\left(\frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} a\right) C^{\frac{1}{\theta}} K^{1-\theta} - R_f K\right] | q, a + e\right\} \tag{3}$$

This leads to

$$K = (1 - \frac{1}{\theta})^{\theta} C \left\{\mathbb{E}\left[\exp\left(\frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} a\right) | q, a + e\right]\right\}^{\theta} \tag{4}$$

with $R_f = 1$. It shows that $K = K(s, q)$.

Investors' problem

- Each investor is indexed by i. They derive utility from end of period wealth

$$U(W_i) = -\exp\left(-\gamma W_i\right),$$

They face the budget constraint:

$$W_i = \left(W_0 - d_i q\right) R_f + d_i v = W_0 + D_i(v - q).$$

- investor i solves

$$-\max_{d_i} \mathbb{E}^i \left\{\exp\left[-\gamma W_0 + D_i(v - q)\right]\right\} \tag{5}$$

where $\mathbb{E}^i$ is expectation operator based on investor i's information.

Equilibrium

The equilbrium price function is a mapping between the signal s, ε and x, such that

1. $q = q(s, \varepsilon, x)$ clears the function market
$$\int_0^1 D_i = x \tag{6}$$
where D_i solves (5)
2. given the price function $q(s, \varepsilon, x)$, investment $K = K(s, q)$ solves firm's (3).
3. We conjecture $q = q(s, \varepsilon, x) = q_0 + \phi_s s + \phi_\varepsilon \varepsilon + \phi_x x$ and $\log K \equiv k = k_0 + \pi_s s + \pi_q q$, where $q_0, \phi_s, \phi_\varepsilon, \phi_x, k_0, \pi_s, \pi_q$ are undetermined coefficients. Notice that q, k are normally distributed random variables

Equilibrium

4. The final payoff

$$v = \frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta}a + \frac{1}{\theta}c + \frac{\theta - 1}{\theta}k$$

   is hence also random variable with normal distribution. Notice that $k = k_0 + \pi_s s + \pi_q q$ and s,and q are public information. So k is known to every investor with certainty.

5. Then by the property of normal distribution

$$\begin{aligned} & -\max_{d_i} \mathbb{E}^i \left\{ \exp\left[-\gamma W_0 - \gamma D_i(v - q) \right] \right\} \\ = \quad & \min_{d_i} \left[-\gamma \left[W_0 + D_i(\mathbb{E}^i v - q) \right] + \frac{\gamma^2}{2} d_i^2 \mathbb{VAR}^i(v) \right] \end{aligned}$$

   or

$$D_i = \frac{\mathbb{E}^i v - q}{\gamma \mathbb{VAR}^i(v)} \quad (7)$$

Asset demand

- Notice

$$\mathbb{E}[\frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} a | a + e, \varepsilon, q] = \frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} \rho_a (a + e)$$

$$\mathbb{VAR}(\frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} a | a + e, \varepsilon, q) = \left(\frac{\theta - 1}{\theta}\right)^2 (1 - \rho_a) \sigma_a^2$$

where $\rho_\alpha = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}$ denotes the informativeness of the signal s.

- So the demand from the informed trader is given by

$$D_I = \frac{\frac{\varepsilon}{\theta} + \frac{\theta-1}{\theta} \rho (a + e) + \frac{1}{\theta} c + \frac{\theta-1}{\theta} k - q}{\gamma \left(\frac{\theta-1}{\theta}\right)^2 (1 - \rho_a) \sigma_a^2}$$

Asset demand

- Notice for the uninformed

$$\mathbb{E}[\frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} a | a + e, q] = \frac{1}{\theta}\mathbb{E}[\varepsilon | q] + \frac{\theta - 1}{\theta}\rho_a (a + e)$$

$$\mathbb{VAR}(\frac{\varepsilon}{\theta} + \frac{\theta - 1}{\theta} a | a + e, q) = \left(\frac{\theta - 1}{\theta}\right)^2 (1 - \rho_a)\sigma_a^2 + \frac{1}{\theta^2}\mathbb{VAR}(\varepsilon | q)$$

where $\mathbb{VAR}(\varepsilon|q)$ will be a constant due to the property of normal distribution, and $\mathbb{E}[\varepsilon|q]$ will depend on q.

- Their demand

$$D_U = \frac{\frac{1}{\theta}\mathbb{E}[\varepsilon|q] + \frac{\theta-1}{\theta}\rho (a + e) + \frac{1}{\theta}c + \frac{\theta-1}{\theta}k - q}{\gamma\left[\left(\frac{\theta-1}{\theta}\right)^2 (1 - \rho)\sigma_a^2 + \frac{1}{\theta^2}\mathbb{VAR}(\varepsilon|q)\right]}$$

Asset Market Equilibrium

- The equilibrium in the asset market requries

$$
\begin{aligned}
x &= \lambda D_I + (1-\lambda) D_U \\
&= \lambda \frac{\frac{\varepsilon}{\theta} + \frac{\theta-1}{\theta}\rho\left(a+e\right) + \frac{1}{\theta}c + \frac{\theta-1}{\theta}k - q}{\gamma\left(\frac{\theta-1}{\theta}\right)^2 (1-\rho)\sigma_a^2} \\
&\quad + (1-\lambda)\frac{\frac{1}{\theta}\mathbb{E}[\varepsilon|q] + \frac{\theta-1}{\theta}\rho\left(a+e\right) + \frac{1}{\theta}c + \frac{\theta-1}{\theta}k - q}{\gamma\left[\left(\frac{\theta-1}{\theta}\right)^2 (1-\rho)\sigma_a^2 + \frac{1}{\theta^2}\mathbb{VAR}(\varepsilon|q)\right]}
\end{aligned}
$$

- Define $\tilde{q} = \frac{\varepsilon}{\theta} - \frac{\gamma\left(\frac{\theta-1}{\theta}\right)^2(1-\rho)\sigma_a^2}{\lambda}x$, an educated conjecture of q takes

$$
\begin{aligned}
q &= \frac{\theta-1}{\theta}\rho\left(a+e\right) + \frac{1}{\theta}c + \frac{\theta-1}{\theta}k \\
&\quad + \text{a linear function of } \tilde{q}
\end{aligned}
$$

The informativeness of the price

- We defined

$$\rho_q = \frac{\left(\frac{1}{\theta}\right)^2 \sigma_\varepsilon^2}{\left(\frac{1}{\theta}\right)^2 \sigma_\varepsilon^2 + \left[\frac{\gamma\left(\frac{\theta-1}{\theta}\right)^2 (1-\rho)\sigma_a^2}{\lambda}\right]^2 \sigma_x^2} \tag{8}$$

as the informativeness of the price. It is easy to see that ρ_q increases with ρ for given λ. As the precision of firm's **DISCLOSURED** information increases, the informed investors trade more aggressively. So price becomes more informative too.

Firms' investment problem

- We now consider the firm's investment.

$$
\begin{aligned}
K &= (1-\frac{1}{\theta})^{\theta} C \left\{ \mathbb{E}\left[\exp\left(\frac{\varepsilon}{\theta} + \frac{\theta-1}{\theta} a \right) |q, a+e \right] \right\}^{\theta} \\
&= (1-\frac{1}{\theta})^{\theta} C \left\{ \mathbb{E}\left[\exp\left(\frac{\varepsilon}{\theta} + \frac{\theta-1}{\theta} a \right) |\tilde{q}, a+e \right] \right\}^{\theta}
\end{aligned}
$$

This leads to

$$
\begin{aligned}
\log K &= k_0 + \theta \rho_q \tilde{q} + (\theta-1)\rho(a+e) \\
&= k_0 + \rho_q \left[\varepsilon - \frac{\gamma\theta \left(\frac{\theta-1}{\theta}\right)^2 (1-\rho)\sigma_a^2}{\lambda} x \right] + (\theta-1)\rho(a+e)
\end{aligned}
$$

Firms' investment problem

- We can compute the expected profit for the firms

$$
\begin{aligned}
\Pi &= \mathbb{E}_{\tilde{q},a+e}\left\{\mathbb{E}\left\{\left[\exp\left(\frac{\varepsilon}{\theta}+\frac{\theta-1}{\theta}a\right)C^{\frac{1}{\theta}}K^{1-\theta}-K\right]|\tilde{q},a+e\right\}\right\} \\
&= \frac{1}{\theta}(1-\frac{1}{\theta})^{\theta-1}C\left\{\mathbb{E}\left[\exp\left(\frac{\varepsilon}{\theta}+\frac{\theta-1}{\theta}a\right)|\tilde{q},a+e\right]\right\}^{\theta} \\
&= \frac{1}{\theta}(1-\frac{1}{\theta})^{\theta-1}C\exp\left[\frac{(\theta-1)(\theta-2)}{2}\sigma_a^2\right]\div \\
&\quad \exp\left[\frac{1}{2}\frac{(\theta-1)^3}{\theta}(1-\rho)\sigma_a^2+\frac{1}{2}\frac{\theta-1}{\theta}(1-\rho_q)\sigma_\varepsilon^2\right]
\end{aligned}
$$

- Notice that the firms' profit increases with both ρ and ρ_q.
  - A more informed signal about a makes investment more aligned with firms' productivity
  - A more informed signal about a make financial price more informed about its demand

Short Summary

So far, we have characterized a partial equilibrium with **exogenous** information. We show

1. equilibrium is unique
2. multiple equilibria may arise with **endogenous** information

Endogenous Information

Now we assume information is endogenously. We add a period 0 to the model. Investors and firms decide whether or not to acquire information simultaneously in period 0.

1. an investor pays φ_I to know ε perfectly (to know ε with noisy works as well)
2. the firm pays φ_F to obtain a signal $s = \varepsilon + e$ with $e \sim \mathcal{N}(0, \sigma_e^2)$ otherwise the firms obtain an useless signal $\tilde{s} = \varepsilon + \tilde{e}$ with $e \sim \mathcal{N}(0, \infty)$.
3. our purpose to understand information acquisition of the firm, information acquisition of financial market, and their interactions.

Firms' information acquisition

1. Firm acquire information if and only if

$$\exp\left[\frac{(\theta-1)^3}{2\theta}\rho\sigma_a^2\right] \geq 1+\frac{\varphi_F \exp\left[\frac{(\theta-1)}{2\theta}\sigma_a^2+\frac{\theta-1}{2\theta}(1-\rho_q)\sigma_\varepsilon^2\right]}{\frac{1}{\theta}(1-\frac{1}{\theta})^{\theta-1}C} \quad (9)$$

2. given other parameters, firm acquire information if and only if

$$\rho_q > \hat{\rho}_q$$

   where $\hat{\rho}_q$ makes the equality holds for (9). An increase in ρ_q provides an accurate information for firm's demand shocks, increase the firms' expected profit and hence better incentive to acquire information about a.

3. It is easy to see $\hat{\rho}_q$ increases with φ_F and decreases with C.

Information acquisition for the financial market

1. We seek an interior solution such that $0 < \lambda < 1$ fraction of investors aquire information.
2. So informed and uninformed investors attain the same expected utility ex ante. This leads to

$$
\begin{aligned}
& \exp(\gamma\varphi_I)\sqrt{\frac{var(\frac{\varepsilon}{\theta}+\frac{\theta-1}{\theta}a|\varepsilon,s)}{var(\frac{\varepsilon}{\theta}+\frac{\theta-1}{\theta}a|\tilde{q},s)}} \\
= & \exp(\gamma\varphi_I)\sqrt{\frac{(\theta-1)^2(1-\rho_a)\sigma_a^2}{(\theta-1)^2(1-\rho_a)\sigma_a^2+\sigma_\varepsilon^2(1-\rho_q)}} \\
= & 1
\end{aligned}
$$

   and $\lambda = \lambda(\rho_q)$ is then determined by (8).
3. We have $\varphi_I \Uparrow \Rightarrow \rho_q \Downarrow$ and $\rho_a \Uparrow \Rightarrow \rho_q \Uparrow$

Intuition

The expected utility of the informed to the uninformed is

$$\frac{EU_I(W)}{EU_U(W)} = \exp(\gamma\varphi_I)\sqrt{\frac{var(\frac{\varepsilon}{\theta}+\frac{\theta-1}{\theta}a|\varepsilon,s)}{var(\frac{\varepsilon}{\theta}+\frac{\theta-1}{\theta}a|\tilde{q},s)}}$$

$$\frac{10000+5}{10000+10} > \frac{1+5}{1+10}$$

Information acquisition in Equilibrium

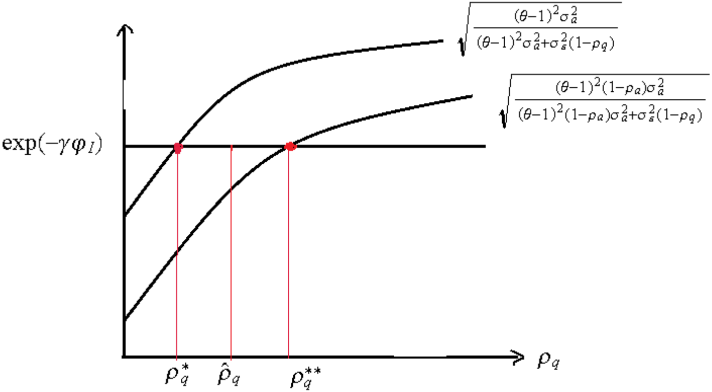
1. Define $\rho_q^* < \rho_q^{**}$ such that

$$1 = \exp(\gamma\varphi_I)\sqrt{\frac{(\theta-1)^2\sigma_a^2}{(\theta-1)^2\sigma_a^2+\sigma_\varepsilon^2(1-\rho_q^*)}}$$

$$1 = \exp(\gamma\varphi_I)\sqrt{\frac{(\theta-1)^2(1-\rho_a)\sigma_a^2}{(\theta-1)^2(1-\rho_a)\sigma_a^2+\sigma_\varepsilon^2(1-\rho_q^{**})}}$$

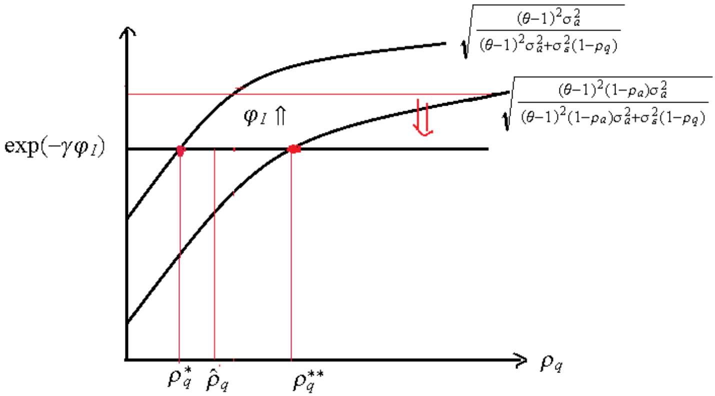
2. if $\rho_q^{**} < \hat{\rho}_q$, unique equilibrium. $\rho_a = 0$ and and $\rho_q = \rho_q^*$.
3. if $\rho_q^* > \hat{\rho}_q$, then unique equilibrium. $\rho_a = \rho_a^*$ $\rho_q = \rho_q^{**}$.
4. if $\rho_q^* < \hat{\rho}_q \leq \rho_q^{**}$ or $\rho_q^* \leq \hat{\rho}_q < \rho_q^{**}$ then two equilibria.

Two equilibria



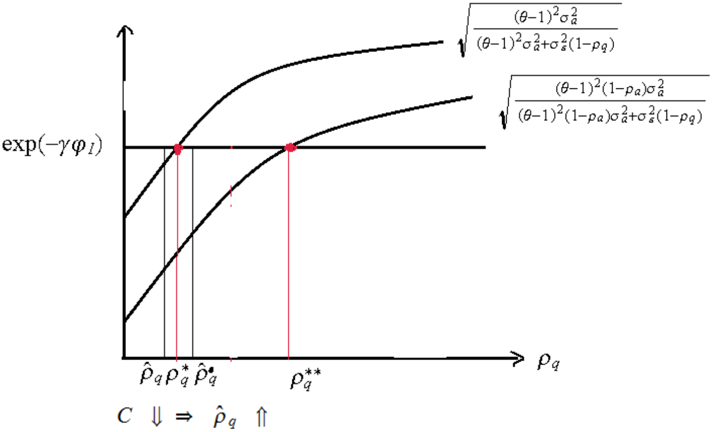
A graphical illustration of two equilibria

A decrease in in information acquisition cost in financial market



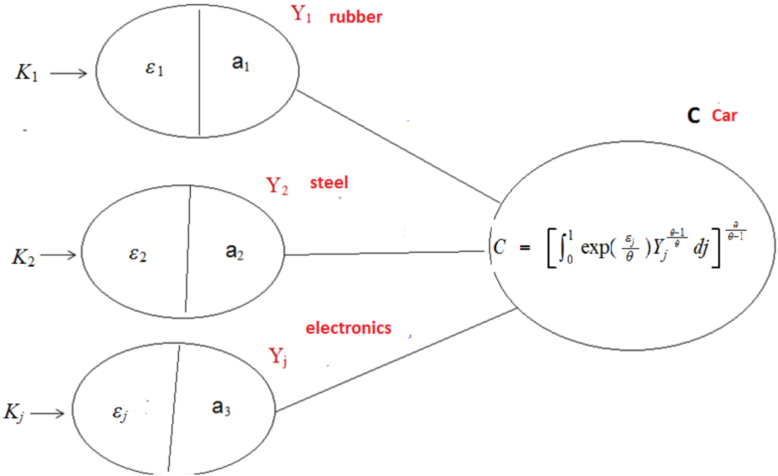
An increase φ_I moves the economy from unique equilbrium to two equilibria.

A decrease in aggregate output



An small decreases in aggregate C can reduce the price efficiency from ρ_q^{**} to ρ_q^*.

Production Structure



Model setup

- A continuum of identical islands. They are linked with trade in goods

$$C = \left[\int \exp(\frac{1}{\theta}\varepsilon_j) Y_j^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{10}$$

The demand for each product j is now

$$Y_j = P_j^{-\theta} C \exp(\frac{1}{\theta}\varepsilon_j)$$

and

$$Y_j = \exp(a_j) K_j$$

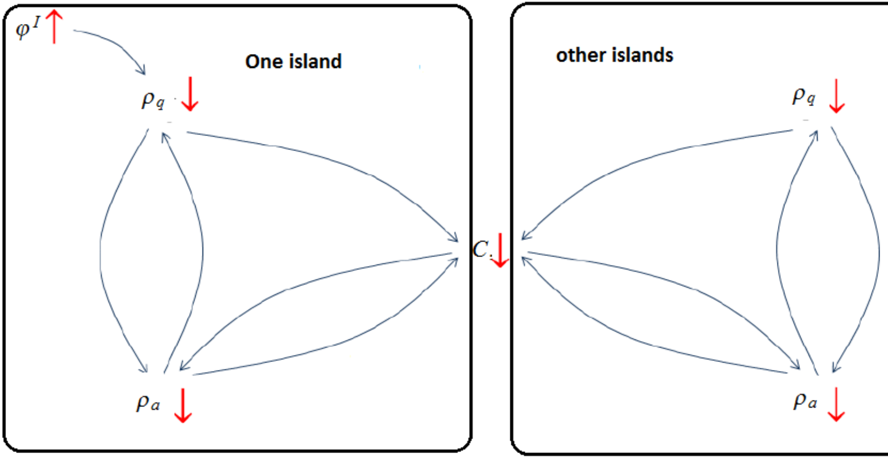
- So given C, equilibrium in each island is as before. The aggregate production is then decided by (10).

Information acquisition complementarity across islands

- Now one additional level of information acquisition complementarity
    - as more firms learn their a_j, aggregate output increases
    - as aggregate output increases, more firms learn their a_j
    - at same time financial prices will be more informative in each island, which trigger another round of informational accelerator

$\Rightarrow$Information contagion

Information contagion



Resource Misallocation

If we consider a symmetric equilibrium, the aggregate TFP is then given

$$A^G\left(\rho_a, \rho_q\right) = \left(\int\left[\mathbb{E}\left(A_j^{\frac{\theta-1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j, x_j^q\right)\right]^\theta dj\right)^{\frac{1}{\theta-1}} \quad (11)$$

$$= \exp\left\{\begin{array}{c} -\frac{1}{2\theta}\sigma_a^2 + \frac{(\theta-1)^2}{2\theta}\rho_a\sigma_a^2 \\ -\frac{1}{2\theta}(1-\rho_q)\sigma_\varepsilon^2 \end{array}\right\},$$

Conclusion

- We develop an Grossman-Stiglitz type model with real investment and aggregate production
- We shows information acquisition in the real economy and financial sector reinforce themselves
- Such a two-way learning mechanism produces important macroeconomic consequences
    - financial market efficiency affects resource misallocation
    - contagion