Stochastic calculus and its applications in quantitative finance

National School of Development, Peking University, Fall 2018

Dates and times: Every Wednesday and every other Thursday (in odd weeks), 3:10 $\sim$ 5:00 pm.

Instructor: Tai-Ho Wang
Coordinates: Email: tai-ho.wang@baruch.cuny.edu

Course Policy: Homework will be assigned every week. No late homework will be accepted unless previous arrangements have been made.
Midterm: April 26, 2018, Thursday, 3:10 $\sim$ 5pm
Final: June 27, 2018, Wednesday, 2 $\sim$ 4pm.

Grading: The course is graded based on homework, midterm exam, and the final exam according to the formula:

$$\text{Grade} \quad = \quad (\text{HW} + \text{Midterm}) \times 30\% + \text{Final} \times 40\%$$

Any student who is absent from the final examination and whose term average is at least 55% must report and present a valid excuse to the Office of National School of Development. If the term average is less than 55%, any student who misses the final exam is given an F grade.

Cheating and plagiarism: There is a zero-tolerance policy on cheating and plagiarism. I fully comply with the policy of National School of Development on Academic Honesty. Anyone caught cheating on an exam will receive a grade F for the course. You are encouraged to discuss the course material and homework assignments with your fellow students. However, each assignment must be written and completed by each individual on his or her own; should two homework submissions be found to be close to identical, both submissions will receive a score of zero. Plagiarism and related forms of unreferenced "borrowing" of written material will not be tolerated. Finally, textbooks, notes, and any other written materials should not be used during the exams unless otherwise noted.

References:

(1) *Probability Essentials* by J. Jacod and P. Protter

(2) *Introduction to stochastic calculus with applications* by F. Klebaner

(3) *Stochastic calculus for finance II: Continuous time models* by S. Shreve

Syllabus[1]

- *Discrete time martingale*
  - Conditional probability and conditional expectation
  - Uniform integrability
  - Stochastic processes in general
  - Martingale, submartingale, supermartingale
  - Doob's inequalities
  - Predictable processes
  - Doob decomposition theorem
  - Stopping times
  - Martingale transformation and discrete stochastic integral
  - Optional stopping theorem
- *Brownian motion*
  - Definition, construction and properties of Brownian motions
  - Reflection principle
  - Brownian motion with drift
  - Cameron-Martin theorem
- *Stochastic calculus*
  - Construction of Itô's integral
  - Martingale property
  - Itô's isometry
  - Quadratic variation
  - Itô's formula
  - Girsanov theorem
  - Stratonovich integral
- *Markov processes*
  - Markov property and strong Markov property

[1]Topics are subject to change.

- Transition matrix, transition density
- Chapman-Kolmogorov equation
- The generator and the infinitesimal generator
- Markov martingales
- Dynkin's formula

▶ *Stochastic differential equations*

- Existence and uniqueness
- Diffusion processes
- Relationship to partial differential equations
- Kolmogorov forward and backward equations
- Feynman-Kac formula

▶ *Stochastic calculus with jump processes*

- Poisson and compound Poisson process
- Itô's formula for processes with jump
- Change of measure for jump processes
- Option pricing in processes with jump

▶ *Numerical stochastic differential equation*

- Itô-Taylor expansion
- Strong and weak convergence
- Variance reduction techniques