数据产权初始分配、私有数据共享与社会福利
——基于“谁投入、谁贡献、谁受益”的产权分配原则

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I.1 代表性家庭

结合代表性家庭的效用最大化问题，构建汉密尔顿（Hamilton）方程：

$$\begin{aligned}\mathbf{H}=&\ln\left[\left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}\right]-\left[\frac{\lambda^c}{N_t^{\,2}}\cdot\kappa\cdot\int_0^{N_t}(x_{it}^c)^2di+\frac{\lambda^c}{N_t}\cdot\tilde{\kappa}\cdot\int_0^{N_t}(\tilde{x}_{it}^c)^2di\right]\\&+\mu_t\cdot\left\{(r_t-n_{\bar{L}})a_t+w_{Lt}(1-l_H)+w_{Ht}l_H+\int_0^{N_t}\left(p_{Dit}^c\cdot\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c-p_{it}\right)\cdot c_{it}di\right\}\\&+\mu_{x0}^c\cdot x_{it}^c+\mu_{x1}^c\cdot(1-x_{it}^c)+\mu_{\tilde{x}0}^c\cdot\tilde{x}_{it}^c+\mu_{\tilde{x}1}^c\cdot(1-\tilde{x}_{it}^c).\end{aligned}$$

可推导得一阶条件包括：

$$\frac{\partial\mathbf{H}}{\partial c_{it}}=c_t^{\frac{1-\sigma}{\sigma}}\cdot c_{it}^{-\frac{1}{\sigma}}+\mu_t\cdot\left[p_{Dit}^c\cdot\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c-p_{it}\right]=0,\tag{I.1}$$

$$\frac{\partial\mathbf{H}}{\partial x_{it}^c}=-\frac{\lambda^c}{N_t^{\,2}}\cdot2\kappa x_{it}^c+\mu_t\cdot p_{Dit}^c\cdot\lambda^c c_{it}+\mu_{x0}^c-\mu_{x1}^c=0,\tag{I.2}$$

$$\frac{\partial\mathbf{H}}{\partial\tilde{x}_{it}^c}=-\frac{\lambda^c}{N_t}\cdot2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}+\mu_{\tilde{x}0}^c-\mu_{\tilde{x}1}^c=0,\tag{I.3}$$

$$\frac{\partial\mathbf{H}}{\partial a_t}=\mu_t\cdot(r_t-n_{\bar{L}})=(\rho-n_{\bar{L}})\mu_t-\dot{\mu}_t.\tag{I.4}$$

由一阶条件式（I.1）可得，

$$\Rightarrow c_t^{\frac{1-\sigma}{\sigma}}\cdot c_{it}^{-\frac{1}{\sigma}}=\mu_t\cdot\left[p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\right],$$

$$\Rightarrow c_t^{\frac{(1-\sigma)^2}{\sigma}}\cdot c_{it}^{\frac{\sigma-1}{\sigma}}=\mu_t^{1-\sigma}\cdot\left[p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\right]^{1-\sigma},$$

$$\Rightarrow c_t^{\frac{(1-\sigma)^2}{\sigma}}\cdot\int_0^{N_t}c_{it}^{\frac{\sigma-1}{\sigma}}di=\mu_t^{1-\sigma}\cdot\int_0^{N_t}\left[p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\right]^{1-\sigma}di,$$

$$\Rightarrow c_t^{\frac{(1-\sigma)^2}{\sigma}+\frac{\sigma-1}{\sigma}}=\mu_t^{1-\sigma}\cdot\int_0^{N_t}\left[p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\right]^{1-\sigma}di,$$

$$\Rightarrow c_t=\frac{1}{\mu_t\cdot\left\{\int_0^{N_t}\left[p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\right]^{1-\sigma}di\right\}^{\frac{1}{1-\sigma}}}.$$

其中，$p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c$ 是考虑家庭出售数据所得收益后的消费品的有效价格。每种消费品的有效价格上升 1 个百分点将引起其消费量下降 σ 个百分点。令标准化 $P_t=\left\{\int_0^{N_t}\left[p_{it}-p_{Dit}^c\cdot\lambda^c x_{it}^c-\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\right]^{1-\sigma}di\right\}^{1/(1-\sigma)}=1$，可得：

$$\Rightarrow c_t=\mu_t^{-1},\tag{I.5}$$

$$\Rightarrow c_t^{\frac{1-\sigma}{\sigma}} \cdot c_{it}^{-\frac{1}{\sigma}} = c_t^{-1} \cdot \left[p_{it} - p_{Dit}^c \cdot \lambda^c x_{it}^c - \tilde{p}_{Dit}^c \cdot \lambda^c \tilde{x}_{it}^c \right],$$

$$\Rightarrow p_{it} - p_{Dit}^c \cdot \lambda^c x_{it}^c - \tilde{p}_{Dit}^c \cdot \lambda^c \tilde{x}_{it}^c = \left(c_t / c_{it}\right)^{\frac{1}{\sigma}} = \left(Y_t / Y_{it}\right)^{\frac{1}{\sigma}}. \tag{I.6}$$

由一阶条件式（I.2），将式（I.5）和对称性条件 $c_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} = N_t^{\frac{\sigma}{\sigma-1}} c_{it}$ 代入后可得，

$$\Rightarrow \mu_t \cdot p_{Dit}^c \cdot \lambda^c c_{it} = \frac{\lambda^c}{N_t^2} \cdot 2\kappa x_{it}^c + \mu_{x1}^c - \mu_{x0}^c,$$

$$\Rightarrow p_{Dit}^c = \left[\frac{2\kappa x_{it}^c}{N_t^2} + \frac{\mu_{x1}^c - \mu_{x0}^c}{\lambda^c} \right] \cdot \frac{c_t}{c_{it}},$$

$$\xrightarrow{\text{对称性}} p_{Dit}^c = \left[2\kappa x_{it}^c + \frac{\mu_{x1}^c - \mu_{x0}^c}{\lambda^c} \cdot N_t^2 \right] \cdot N_t^{\frac{1}{\sigma-1}-1}. \tag{I.7}$$

由一阶条件式（I.3），同理可得，

$$\Rightarrow \mu_t \cdot \tilde{p}_{Dit}^c \cdot \lambda^c c_{it} = \frac{\lambda^c}{N_t} \cdot 2\tilde{\kappa} \tilde{x}_{it}^c + \mu_{\tilde{x}1}^c - \mu_{\tilde{x}0}^c,$$

$$\Rightarrow \tilde{p}_{Dit}^c = \left[\frac{2\tilde{\kappa} \tilde{x}_{it}^c}{N_t} + \frac{\mu_{\tilde{x}1}^c - \mu_{\tilde{x}0}^c}{\lambda^c} \right] \cdot \frac{c_t}{c_{it}},$$

$$\xrightarrow{\text{对称性}} \tilde{p}_{Dit}^c = \left[2\tilde{\kappa} \tilde{x}_{it}^c + \frac{\mu_{\tilde{x}1}^c - \mu_{\tilde{x}0}^c}{\lambda^c} \cdot N_t \right] \cdot N_t^{\frac{1}{\sigma-1}}. \tag{I.8}$$

式（I.7）和式（I.8）是第 i 种产品的消费者数据的供给函数，表明家庭的数据销售价格（p_{Dit}^c 和 $\tilde{p}_{Dit}^c$）与数据出售量的每单位比例（x_{it}^c 和 $\tilde{x}_{it}^c$）正相关，与隐私成本强度（κ 和 $\tilde{\kappa}$）正相关。从市场规模变量关系来看，家庭因承受“大范围”信息泄露而设定的数据价格 $\tilde{p}_{Dit}^c$ 是“小范围”时的 p_{Dit}^c 的 N_t 倍。

由一阶条件式（I.4），将式（I.5）代入后，可得欧拉方程：

$$\Rightarrow \frac{\dot{\mu}_t}{\mu_t} = -(r_t - \rho),$$

$$\Rightarrow g_c = \frac{\dot{c}_t}{c_t} = -\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho. \tag{I.9}$$

式（I.9）是消费的欧拉方程（Euler Equation），表明高利率 r_t（储蓄的未来回报率更高）和低时间贴现率 ρ（对未来消费的重视程度更高）将致使消费者减少当期消费以增加未来消费。

I.2 数据中介

结合第一家数据中介的利润函数，构建拉格朗日（Lagrangean）方程：

$$\mathbf{L}=(1+s_D^c)\tilde{p}_{Dt}^{mc}\cdot\int_0^{N_t}\tilde{D}_t^c di-\int_0^{N_t}\tilde{p}_{Dit}^c\cdot\tilde{x}_{it}^c\cdot\lambda^c R_{it}di+\varphi_t^c\cdot\left[\int_0^{N_t}\left(\tilde{x}_{it}^c\cdot\lambda^c R_{it}\right)di-\tilde{D}_t^c\right].$$

其中，$s_D^c=0$ 表示消费者数据交易没有政府补贴的情况。可推导得一阶条件为：

$$\frac{\partial\mathbf{L}}{\partial[\lambda^c\tilde{x}_{it}^c\cdot R_{it}]}=-\tilde{p}_{Dit}^c+\varphi_t^c=0,$$

$$\Rightarrow\varphi_t^c=\tilde{p}_{Dit}^c.$$

再结合数据中介的零利润条件、紧约束条件、对称性条件 $B_t^c=N_t(\tilde{x}_{it}^c\cdot\lambda^c R_{it})$ 和式(I.8)，可得，

$$(1+s_D^c)\tilde{p}_{Dt}^{mc}\cdot\int_0^{N_t}\tilde{D}_t^c di-\int_0^{N_t}\tilde{p}_{Dit}^c\cdot\tilde{x}_{it}^c\cdot\lambda^c R_{it}di=0,$$

$$\Rightarrow(1+s_D^c)\tilde{p}_{Dt}^{mc}\cdot N_t B_t^c-N_t\tilde{p}_{Dit}^c\cdot\tilde{x}_{it}^c\cdot\lambda^c R_{it}=0,$$

$$\Rightarrow\tilde{p}_{Dt}^{mc}=\frac{\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c\cdot R_{it}N_t}{(1+s_D^c)B_t^c N_t}=\frac{\tilde{p}_{Dit}^c}{(1+s_D^c)N_t},$$

$$\Rightarrow\tilde{p}_{Dt}^{mc}=\frac{\tilde{p}_{Dit}^c}{(1+s_D^c)N_t}=\frac{1}{1+s_D^c}\cdot\left[2\tilde{\kappa}\tilde{x}_{it}^c+\frac{\mu_{\tilde{x}1}^c-\mu_{\tilde{x}0}^c}{\lambda^c}\cdot N_t\right]\cdot N_t^{\frac{1}{\sigma-1}-1}.\tag{I.10}$$

结合第二家数据中介的利润函数，构建拉格朗日（Lagrangean）方程：

$$\mathbf{L}=(1+s_D^f)\tilde{p}_{Dt}^{mf}\cdot\int_0^{N_t}\tilde{D}_t^f di-\int_0^{N_t}\tilde{p}_{Dit}^f\cdot\tilde{x}_{it}^f\cdot(1-\lambda^c)R_{it}di+\varphi_t^f\cdot\left\{\int_0^{N_t}\left[\tilde{x}_{it}^f\cdot(1-\lambda^c)R_{it}\right]di-\tilde{D}_t^f\right\}.$$

其中，$s_D^f=0$ 表示企业数据交易没有政府补贴的情况。可推导得一阶条件为：

$$\frac{\partial\mathbf{L}}{\partial\left[(1-\lambda^c)\tilde{x}_{it}^f\cdot R_{it}\right]}=-\tilde{p}_{Dit}^f+\varphi_t^f=0,$$

$$\Rightarrow\varphi_t^f=\tilde{p}_{Dit}^f.$$

再结合数据中介的零利润条件、紧约束条件、对称性条件 $B_t^f=N_t[\tilde{x}_{it}^f\cdot(1-\lambda^c)R_{it}]$，可得，

$$(1+s_D^f)\tilde{p}_{Dt}^{mf}\cdot\int_0^{N_t}\tilde{D}_t^f di-\int_0^{N_t}\tilde{p}_{Dit}^f\cdot\tilde{x}_{it}^f\cdot(1-\lambda^c)R_{it}di=0,$$

$$\Rightarrow(1+s_D^f)\tilde{p}_{Dt}^{mf}\cdot N_t B_t^f-N_t\tilde{p}_{Dit}^f\cdot\tilde{x}_{it}^f\cdot(1-\lambda^c)R_{it}=0,$$

$$\Rightarrow\tilde{p}_{Dt}^{mf}=\frac{\tilde{p}_{Dit}^f\cdot(1-\lambda^c)\tilde{x}_{it}^f\cdot R_{it}N_t}{(1+s_D^f)B_t^f N_t}=\frac{\tilde{p}_{Dit}^f}{(1+s_D^f)N_t}.\tag{I.11}$$

I.3 代表性企业

结合代表性企业价值最大化问题，构建拉格朗日（Lagrangean）方程：

$$\mathbf{L}=\left\{\begin{array}{l}(1-\tau_Y)\left[p_{Dit}^c\cdot\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c+\left(Y_t/Y_{it}\right)^{1/\sigma}\right]\cdot Y_{it}-w_{Lt}L_{yit}-w_{Ht}H_{yit}-\tilde{p}_{Dt}^{mc}\cdot\tilde{D}_t^c\\-\tilde{p}_{Dt}^{mf}\cdot\tilde{D}_t^f-p_{Dit}^c\cdot D_{it}^c+\tilde{p}_{Dit}^f\cdot\tilde{x}_{it}^f\cdot(1-\lambda^c)Y_{it}+\dot{V}_{it}-\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2\cdot V_{it}\\+\mu_{x0}^f\cdot x_{it}^f+\mu_{x1}^f\cdot(1-x_{it}^f)+\mu_{\tilde{x}0}^f\cdot\tilde{x}_{it}^f+\mu_{\tilde{x}1}^f\cdot(1-\tilde{x}_{it}^f)\end{array}\right\}.$$

其中，$\tau_Y=0$ 表示没有政府税收的情况。可推导得一阶条件包括：

$$\frac{\partial \mathbf{L}}{\partial L_{yit}}=\frac{\partial \mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial L_{yit}}-w_{Lt}=0, \tag{I. 12}$$

$$\frac{\partial \mathbf{L}}{\partial H_{yit}}=\frac{\partial \mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial H_{yit}}-w_{Ht}=0, \tag{I. 13}$$

$$\frac{\partial \mathbf{L}}{\partial \tilde{D}_t^c}=\frac{\partial \mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial \tilde{D}_t^c}-\tilde{p}_{Dt}^{mc}=0, \tag{I. 14}$$

$$\frac{\partial \mathbf{L}}{\partial \tilde{D}_t^f}=\frac{\partial \mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial \tilde{D}_t^f}-\tilde{p}_{Dt}^{mf}=0, \tag{I. 15}$$

$$\frac{\partial \mathbf{L}}{\partial D_{it}^c}=\frac{\partial \mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial D_{it}^c}-p_{Dit}^{c}=0, \tag{I. 16}$$

$$\frac{\partial \mathbf{L}}{\partial x_{it}^f}=\frac{\partial \mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial x_{it}^f}+\mu_{x0}^f-\mu_{x1}^f=0, \tag{I. 17}$$

$$\frac{\partial \mathbf{L}}{\partial \tilde{x}_{it}^f}=\tilde{p}_{Dit}^f\cdot(1-\lambda^c)Y_{it}-2\delta(1-\lambda^c)\tilde{x}_{it}^f\cdot V_{it}+\mu_{\tilde{x}0}^f-\mu_{\tilde{x}1}^f=0. \tag{I. 18}$$

由一阶条件式（I. 12），继续推导：

$$\frac{\partial \mathbf{L}}{\partial Y_{it}}=(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^c\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\lambda^c\tilde{x}_{it}^c\right]+\tilde{p}_{Dit}^f\cdot(1-\lambda^c)\tilde{x}_{it}^f.$$

结合生产函数得，

$$\Rightarrow\frac{\partial Y_{it}}{\partial L_{yit}}=\left\{\begin{array}{l}\dfrac{(1-\eta)Y_{it}\cdot\dfrac{\zeta}{\zeta-1}\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}-1}\cdot\dfrac{\zeta-1}{\zeta}(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}-1}z_L}{\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}}\\ +\dfrac{\eta Y_{it}\cdot\alpha(1-\lambda^c)x_{it}^f}{\alpha\left[D_{it}^c+(1-\lambda^c)x_{it}^f\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^c+\tilde{D}_t^f)}\cdot\dfrac{\partial Y_{it}}{\partial L_{yit}}\end{array}\right\},$$

$$\Rightarrow\frac{\partial Y_{it}}{\partial L_{yit}}=\frac{(1-\eta)Y_{it}\cdot z_L^{\frac{\zeta-1}{\zeta}}(L_{yit})^{\frac{\zeta-1}{\zeta}-1}}{\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]\cdot\left[1-\dfrac{\eta Y_{it}\cdot\alpha(1-\lambda^c)x_{it}^f}{\alpha\left[D_{it}^c+(1-\lambda^c)x_{it}^f\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^c+\tilde{D}_t^f)}\right]}.$$

代入一阶条件式（I. 12）可得，

$$\frac{(1-\eta)Y_{it}\cdot z_L^{\frac{\zeta-1}{\zeta}}L_{yit}^{\frac{\zeta-1}{\zeta}-1}\cdot\left\{(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^c\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\lambda^c\tilde{x}_{it}^c\right]+\tilde{p}_{Dit}^f\cdot(1-\lambda^c)\tilde{x}_{it}^f\right\}}{\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]\cdot\left[1-\dfrac{\eta Y_{it}\cdot\alpha(1-\lambda^c)x_{it}^f}{\alpha\left[D_{it}^c+(1-\lambda^c)x_{it}^f\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^c+\tilde{D}_t^f)}\right]}=w_{Lt}. \tag{I. 19}$$

同理，由一阶条件式（I. 13）至式（I. 16）可得，

$$\frac{(1-\eta)Y_{it}\cdot z_H^{\frac{\zeta-1}{\zeta}}H_{yit}^{\frac{\zeta-1}{\zeta}-1}\cdot\left\{(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^{c}\lambda^{c}x_{it}^{c}+\tilde{p}_{Dit}^{c}\lambda^{c}\tilde{x}_{it}^{c}\right]+\tilde{p}_{Dit}^{f}\cdot(1-\lambda^{c})\tilde{x}_{it}^{f}\right\}}{\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]\cdot\left[1-\frac{\eta Y_{it}\cdot\alpha(1-\lambda^{c})x_{it}^{f}}{\alpha\left[D_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^{c}+\tilde{D}_t^{f})}\right]}=w_{Ht},\tag{I.20}$$

$$\frac{(1-\alpha)\eta Y_{it}\cdot\left\{(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^{c}\lambda^{c}x_{it}^{c}+\tilde{p}_{Dit}^{c}\lambda^{c}\tilde{x}_{it}^{c}\right]+\tilde{p}_{Dit}^{f}\cdot(1-\lambda^{c})\tilde{x}_{it}^{f}\right\}}{\alpha\left[D_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^{c}+\tilde{D}_t^{f})-\eta Y_{it}\cdot\alpha(1-\lambda^{c})x_{it}^{f}}=\tilde{p}_{Dt}^{mc},\tag{I.21}$$

$$\frac{(1-\alpha)\eta Y_{it}\cdot\left\{(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^{c}\lambda^{c}x_{it}^{c}+\tilde{p}_{Dit}^{c}\lambda^{c}\tilde{x}_{it}^{c}\right]+\tilde{p}_{Dit}^{f}\cdot(1-\lambda^{c})\tilde{x}_{it}^{f}\right\}}{\alpha\left[D_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^{c}+\tilde{D}_t^{f})-\eta Y_{it}\cdot\alpha(1-\lambda^{c})x_{it}^{f}}=\tilde{p}_{Dt}^{mf},\tag{I.22}$$

$$\frac{\alpha\eta Y_{it}\cdot\left\{(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^{c}\lambda^{c}x_{it}^{c}+\tilde{p}_{Dit}^{c}\lambda^{c}\tilde{x}_{it}^{c}\right]+\tilde{p}_{Dit}^{f}\cdot(1-\lambda^{c})\tilde{x}_{it}^{f}\right\}}{\alpha\left[D_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^{c}+\tilde{D}_t^{f})-\eta Y_{it}\cdot\alpha(1-\lambda^{c})x_{it}^{f}}=p_{Dit}^{c}.\tag{I.23}$$

结合式（I.19）至式（I.23）可得，

$$\tilde{p}_{Dt}^{mc}=\tilde{p}_{Dt}^{mf}=\frac{1-\alpha}{\alpha}\cdot p_{Dit}^{c},\tag{I.24}$$

$$\alpha\left[D_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^{c}+\tilde{D}_t^{f})=\frac{(1-\alpha)\eta}{1-\eta}\cdot\frac{w_{Lt}L_{yit}+w_{Ht}H_{yit}}{\tilde{p}_{Dt}^{mf}}.\tag{I.25}$$

由一阶条件式（I.17）可得，企业无成本地完全使用其自身拥有的内部数据，也即：

$$\Rightarrow\mu_{x1}^{f}-\mu_{x0}^{f}=\frac{\partial\mathbf{L}}{\partial Y_{it}}\cdot\frac{\partial Y_{it}}{\partial x_{it}^{f}}>0,$$

$$\Rightarrow\quad\mu_{x1}^{f}>0\quad\Rightarrow\quad x_{it}^{f*}=\max x_{it}^{f}=1.\tag{I.26}$$

对宏观经济总产出或社会福利而言，当经济体的市场规模足够大（$N_t\to\infty$）时，相较于所有企业共享共用社会外部数据 $\tilde{D}_t^{c}+\tilde{D}_t^{f}$ 所带来的巨大经济效益，企业使用内部数据 $(1-\lambda^{c})x_{it}^{f}\cdot Y_{it}+D_{it}^{c}$ 所带来的经济效益微乎其微。也即，企业对内部数据的使用情况将不影响社会外部数据共享程度、劳动力配置情况和经济总量。据此，式（I.24）和式（I.25）联立易得 $(\tilde{p}_{Dt}^{mc}\cdot\tilde{D}_t^{c}+\tilde{p}_{Dt}^{mf}\cdot\tilde{D}_t^{f})/(w_{Lt}L_{yit}+w_{Ht}H_{yit})=\eta/(1-\eta)$，这符合柯布—道格拉斯(Cobb-Douglas）形式生产函数的要素收入分配的份额关系式。

I.4 经济均衡

I.4.1 求解经济均衡

由式（I.24），将式（I.7）和式（I.10）代入可得，

$$\Rightarrow \frac{1-\alpha}{\alpha}=\frac{\tilde{p}_{Dt}^{mc}}{p_{Dit}^{c}}=\frac{\frac{1}{1+s_D^c}\cdot\left[2\tilde{\kappa}\tilde{x}_{it}^{c}+\frac{\mu_{\tilde{x}1}^{c}-\mu_{\tilde{x}0}^{c}}{\lambda^c}\cdot N_t\right]\cdot N_t^{\frac{1}{\sigma-1}-1}}{\left[2\kappa x_{it}^{c}+\frac{\mu_{x1}^{c}-\mu_{x0}^{c}}{\lambda^c}\cdot N_t^{2}\right]\cdot N_t^{\frac{1}{\sigma-1}-1}},$$

$$\Rightarrow x_{it}^{c}=\frac{\alpha}{(1+s_D^c)(1-\alpha)}\left[\frac{\tilde{\kappa}}{\kappa}\cdot\tilde{x}_{it}^{c}+\frac{\mu_{\tilde{x}1}^{c}-\mu_{\tilde{x}0}^{c}}{2\kappa\lambda^c}\cdot N_t\right]-\frac{\mu_{x1}^{c}-\mu_{x0}^{c}}{2\kappa\lambda^c}\cdot N_t^{2}. \tag{I.27}$$

由式（I.21），将经济均衡时的数据市场出清条件、式（I.10）、式（I.11）、式（I.24）以及 $(Y_t/Y_{it})^{\frac{1}{\sigma}}=N_t^{\frac{1}{\sigma-1}}$ 代入，可得，

$$\Rightarrow \alpha\left[D_{it}^{c}+(1-\lambda^c)x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^{c}+\tilde{D}_t^{f})-\eta Y_{it}\cdot\alpha(1-\lambda^c)x_{it}^{f}$$
$$=\frac{(1-\alpha)\eta\cdot Y_{it}}{\tilde{p}_{Dt}^{mc}}\cdot\left\{(1-\tau_Y)\left[\left(1-\frac{1}{\sigma}\right)\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}+p_{Dit}^{c}\lambda^c x_{it}^{c}+\tilde{p}_{Dit}^{c}\lambda^c\tilde{x}_{it}^{c}\right]+\tilde{p}_{Dit}^{f}\cdot(1-\lambda^c)\tilde{x}_{it}^{f}\right\},$$

$$\Rightarrow \alpha\left[\lambda^c x_{it}^{c}\cdot Y_{it}+(1-\lambda^c)x_{it}^{f}\cdot Y_{it}\right]+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^{c}\cdot Y_{it}N_t+(1-\lambda^c)\tilde{x}_{it}^{f}\cdot Y_{it}N_t\right]-\eta Y_{it}\cdot\alpha(1-\lambda^c)x_{it}^{f}$$
$$=\frac{(1-\alpha)\eta\cdot Y_{it}}{\tilde{p}_{Dt}^{mc}}\cdot\left\{\begin{array}{l}(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)N_t^{\frac{1}{\sigma-1}}+(1-\tau_Y)\frac{\alpha}{1-\alpha}\cdot\tilde{p}_{Dt}^{mc}\lambda^c x_{it}^{c}\cdot\\+(1-\tau_Y)(1+s_D^c)\cdot\tilde{p}_{Dt}^{mc}\lambda^c\tilde{x}_{it}^{c}\cdot N_t+(1+s_D^f)\cdot\tilde{p}_{Dt}^{mc}(1-\lambda^c)\tilde{x}_{it}^{f}\cdot N_t\end{array}\right\},$$

$$\Rightarrow \alpha\left[\lambda^c x_{it}^{c}+(1-\eta)(1-\lambda^c)x_{it}^{f}\right]\cdot N_t^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^{c}+(1-\lambda^c)\tilde{x}_{it}^{f}\right]$$
$$=(1-\alpha)\eta\cdot\left\{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)\frac{N_t^{\frac{1}{\sigma-1}-1}}{\tilde{p}_{Dt}^{mc}}+(1-\tau_Y)\frac{\alpha}{1-\alpha}\cdot\lambda^c x_{it}^{c}\cdot N_t^{-1}+(1-\tau_Y)(1+s_D^c)\cdot\lambda^c\tilde{x}_{it}^{c}+(1+s_D^f)\cdot(1-\lambda^c)\tilde{x}_{it}^{f}\right\},$$

$$\xrightarrow{N_t\to\infty}\frac{\tilde{p}_{Dt}^{mc}}{N_t^{\frac{1}{\sigma-1}-1}}=\frac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}}=\frac{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)}{\left[\frac{1}{\eta}-(1-\tau_Y)(1+s_D^c)\right]\cdot\lambda^c\tilde{x}_{it}^{c}+\left[\frac{1}{\eta}-(1+s_D^f)\right]\cdot(1-\lambda^c)\tilde{x}_{it}^{f}}. \tag{I.28}$$

联立式（I.10）和式（I.28）可得，

$$\frac{2\tilde{\kappa}\tilde{x}_{it}^{c}+\frac{\mu_{\tilde{x}1}^{c}-\mu_{\tilde{x}0}^{c}}{\lambda^c}\cdot N_t}{1+s_D^c}=\frac{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)}{\left[\frac{1}{\eta}-(1-\tau_Y)(1+s_D^c)\right]\cdot\lambda^c\tilde{x}_{it}^{c}+\left[\frac{1}{\eta}-(1+s_D^f)\right]\cdot(1-\lambda^c)\tilde{x}_{it}^{f}}. \tag{I.29}$$

其中，式（I.29）的左侧可表示共享消费者数据的边际成本，右侧可表示共享消费者数据的边际收益，且实现边际成本等于边际收益。

接着，由代表性企业价值方程可得，

$$r_tV_{it}+\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^{f})^2\cdot V_{it}-\dot{V}_{it}=\left\{\begin{array}{l}(1-\tau_Y)\left[p_{Dit}^{c}\cdot\lambda^c x_{it}^{c}+\tilde{p}_{Dit}^{c}\cdot\lambda^c\tilde{x}_{it}^{c}+\left(Y_t/Y_{it}\right)^{1/\sigma}\right]\cdot Y_{it}-w_{Lt}L_{yit}\\-w_{Ht}H_{yit}-\tilde{p}_{Dt}^{mc}\cdot\tilde{D}_t^{c}-\tilde{p}_{Dt}^{mf}\cdot\tilde{D}_t^{f}-p_{Dit}^{c}\cdot D_{it}^{c}+\tilde{p}_{Dit}^{f}\cdot\tilde{x}_{it}^{f}\cdot(1-\lambda^c)Y_{it}\end{array}\right\}.$$

将数据市场出清条件、式（I.10）、式（I.11）、式（I.24）、式（I.25）、式（I.28）以及$(Y_t/Y_{it})^{\frac{1}{\sigma}}=N_t^{\frac{1}{\sigma-1}}$代入，可得，

$$r_tV_{it}+\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2\cdot V_{it}-\dot{V}_{it}=\left\{\begin{array}{l}(1-\tau_Y)\left[\frac{\alpha}{1-\alpha}\tilde{p}_{Dt}^{mf}\cdot\lambda^c x_{it}^c+(1+s_D^c)\tilde{p}_{Dt}^{mf}N_t\cdot\lambda^c\tilde{x}_{it}^c+N_t^{\frac{1}{\sigma-1}}\right]\cdot Y_{it}\\-\frac{1-\eta}{(1-\alpha)\eta}\left\{\begin{array}{l}\alpha\left[\lambda^c x_{it}^c\cdot Y_{it}+(1-\lambda^c)x_{it}^f\cdot Y_{it}\right]\\+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c\cdot Y_{it}N_t+(1-\lambda^c)\tilde{x}_{it}^f\cdot Y_{it}N_t\right]\end{array}\right\}\cdot\tilde{p}_{Dt}^{mf}\\-\tilde{p}_{Dt}^{mf}\cdot\lambda^c\tilde{x}_{it}^c\cdot Y_{it}N_t-\tilde{p}_{Dt}^{mf}\cdot(1-\lambda^c)\tilde{x}_{it}^f\cdot Y_{it}N_t-\frac{\alpha}{1-\alpha}\tilde{p}_{Dt}^{mf}\cdot\lambda^c x_{it}^c\cdot Y_{it}\\+(1+s_D^f)\tilde{p}_{Dt}^{mf}\cdot(1-\lambda^c)\tilde{x}_{it}^f\cdot Y_{it}N_t\end{array}\right\},$$

$$\frac{r_tV_{it}+\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2\cdot V_{it}-\dot{V}_{it}}{Y_{it}N_t^{\frac{1}{\sigma-1}}}=(1-\tau_Y)+\left\{\begin{array}{l}\left[(1-\tau_Y)(1+s_D^c)-\frac{1}{\eta}\right]\cdot\lambda^c\tilde{x}_{it}^c+\left[(1+s_D^f)-\frac{1}{\eta}\right](1-\lambda^c)\tilde{x}_{it}^f\\-\frac{\alpha}{1-\alpha}\left[\left(\tau_Y+\frac{1-\eta}{\eta}\right)\cdot\lambda^c x_{it}^c+\frac{1-\eta}{\eta}\cdot(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}\end{array}\right\}\cdot\frac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}},$$

$$\xrightarrow{N_t\to\infty}\frac{r_tV_{it}+\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2\cdot V_{it}-\dot{V}_{it}}{Y_{it}N_t^{\frac{1}{\sigma-1}}}=\frac{1-\tau_Y}{\sigma},$$

$$\Rightarrow V_{it}=\frac{(1-\tau_Y)/\sigma}{r_t+\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2-\dot{V}_{it}/V_{it}}\cdot Y_{it}N_t^{\frac{1}{\sigma-1}}.\qquad(\text{I}.30)$$

判断稳态时$\frac{\dot{V}_{it}}{V_{it}}$是常数，结合式（I.9）和$Y_{it}=Y_t\cdot N_t^{-\frac{\sigma}{\sigma-1}}$，稳态时需满足

$$\Rightarrow\frac{\dot{V}_{it}}{V_{it}}=g_{Yi}+\frac{1}{\sigma-1}g_N=g_Y-\frac{\sigma}{\sigma-1}g_N+\frac{1}{\sigma-1}g_N=g_Y-g_N=g_c+(n_{\bar{L}}-g_N),$$

$$\Rightarrow\frac{\dot{V}_{it}}{V_{it}}=r_t-\rho+(n_{\bar{L}}-g_N).\qquad(\text{I}.31)$$

由式（I.18），将式（I.11）、式（I.30）、式（I.31）代入可得，

$$\Rightarrow(1+s_D^f)\tilde{p}_{Dt}^{mf}\cdot(1-\lambda^c)Y_{it}N_t-2\delta(1-\lambda^c)\tilde{x}_{it}^f\cdot V_{it}+\mu_{\tilde{x}0}^f-\mu_{\tilde{x}1}^f=0,$$

$$\Rightarrow\frac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}}=\frac{2\delta(1-\lambda^c)\tilde{x}_{it}^f\cdot V_{it}+\mu_{\tilde{x}1}^f-\mu_{\tilde{x}0}^f}{(1+s_D^f)(1-\lambda^c)Y_{it}N_t^{\frac{1}{\sigma-1}}},$$

$$\Rightarrow\frac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}}=\frac{2\delta\tilde{x}_{it}^f}{(1+s_D^f)}\cdot\frac{(1-\tau_Y)/\sigma}{\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+\rho+g_N-n_{\bar{L}}}+\frac{\mu_{\tilde{x}1}^f-\mu_{\tilde{x}0}^f}{(1+s_D^f)(1-\lambda^c)Y_{it}N_t^{\frac{1}{\sigma-1}}}.\qquad(\text{I}.32)$$

联立式（I.28）和式（I.32）可得，

$$\Rightarrow\left[\begin{array}{l}\frac{1-\tau_Y}{1+s_D^f}\cdot\frac{2\delta\tilde{x}_{it}^f/\sigma}{\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+\rho+g_N-n_{\bar{L}}}\\+\frac{\mu_{\tilde{x}1}^f-\mu_{\tilde{x}0}^f}{(1+s_D^f)(1-\lambda^c)Y_{it}N_t^{\frac{1}{\sigma-1}}}\end{array}\right]=\frac{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)}{\left[\frac{1}{\eta}-(1-\tau_Y)(1+s_D^c)\right]\cdot\lambda^c\tilde{x}_{it}^c+\left[\frac{1}{\eta}-(1+s_D^f)\right]\cdot(1-\lambda^c)\tilde{x}_{it}^f}.\qquad(\text{I}.33)$$

其中，式（I.33）的左侧可表示共享企业数据的边际成本，右侧可表示共享企业数据的边际收益，且实现边际成本等于边际收益。

因此，$\tilde{x}_{it}^{c}=\tilde{x}_{it}^{c*}$ 和 $\tilde{x}_{it}^{f}=\tilde{x}_{it}^{f*}$ 将由式（I.29）与式（I.33）的等式结果计算得到。同时，$x_{it}^{c}=x_{it}^{c*}$ 和 $x_{it}^{f}=x_{it}^{f*}$ 将由式（I.27）与式（I.26）计算得到。

I.4.2 均衡结果 $\left(\tilde{x}_{it}^{c},\tilde{x}_{it}^{f}\right)=\left(\tilde{x}_{it}^{c*},\tilde{x}_{it}^{f*}\right)$ 是内点解

此处考虑 $\left(\tilde{x}_{it}^{c},\tilde{x}_{it}^{f}\right)=\left(\tilde{x}_{it}^{c*},\tilde{x}_{it}^{f*}\right)$ 是内点解的情况， 也即满足条件：

$$\tilde{x}_{it}^{c*}\in(0,1),\quad \mu_{\tilde{x}0}^{c}=\mu_{\tilde{x}1}^{c}=0,$$
$$\tilde{x}_{it}^{f*}\in(0,1),\quad \mu_{\tilde{x}0}^{f}=\mu_{\tilde{x}1}^{f}=0.$$

代入式（I.29）和式（I.33），可得简化为：

$$\frac{2\tilde{\kappa}\tilde{x}_{it}^{c}}{1+s_{D}^{c}}=\frac{1-\tau_{Y}}{1+s_{D}^{f}}\cdot\frac{2\delta\tilde{x}_{it}^{f}/\sigma}{\delta(1-\lambda^{c})\cdot(\tilde{x}_{it}^{f})^{2}+\rho+g_{N}-n_{\bar{L}}}=\frac{(1-\tau_{Y})\left(1-\dfrac{1}{\sigma}\right)}{\left[\dfrac{1}{\eta}-(1-\tau_{Y})(1+s_{D}^{c})\right]\cdot\lambda^{c}\tilde{x}_{it}^{c}+\left[\dfrac{1}{\eta}-(1+s_{D}^{f})\right]\cdot(1-\lambda^{c})\tilde{x}_{it}^{f}},$$

$$\Rightarrow\tilde{x}_{it}^{f}=\frac{(1-\tau_{Y})(1+s_{D}^{c})\left(1-\dfrac{1}{\sigma}\right)}{\left[\dfrac{1}{\eta}-(1+s_{D}^{f})\right]\cdot(1-\lambda^{c})2\tilde{\kappa}\tilde{x}_{it}^{c}}-\frac{\left[\dfrac{1}{\eta}-(1-\tau_{Y})(1+s_{D}^{c})\right]\cdot\lambda^{c}\tilde{x}_{it}^{c}}{\left[\dfrac{1}{\eta}-(1+s_{D}^{f})\right]\cdot(1-\lambda^{c})},$$

$$\Rightarrow\frac{\eta(\sigma-1)}{\left(\dfrac{1}{1+s_{D}^{f}}-\eta\right)\cdot2\sigma\tilde{\kappa}\tilde{x}_{it}^{c}}-\frac{\left[\dfrac{1}{(1-\tau_{Y})(1+s_{D}^{c})}-\eta\right]\cdot\lambda^{c}\tilde{x}_{it}^{c}}{\dfrac{1}{1+s_{D}^{f}}-\eta}=\frac{1}{2\sigma\tilde{\kappa}\tilde{x}_{it}^{c}}-\sqrt{\frac{1}{\left(2\sigma\tilde{\kappa}\tilde{x}_{it}^{c}\right)^{2}}-\frac{(\rho+g_{N}-n_{\bar{L}})(1-\lambda^{c})(1+s_{D}^{f})^{2}}{\delta(1-\tau_{Y})^{2}(1+s_{D}^{c})^{2}}}.\quad\text{(I.34)}$$

I.4.3 均衡结果 $\tilde{x}_{it}^{c}=\tilde{x}_{it}^{c*}$ 是角点解

考虑均衡结果 $\tilde{x}_{it}^{c}=\tilde{x}_{it}^{c*}$ 是角点解的情况。基于式（I.34）计算所得的 $\tilde{x}_{it}^{c*}$，易得式（I.3）中满足：

$$\left.\left\{-\frac{\lambda^{c}}{N_{t}}\cdot2\tilde{\kappa}\tilde{x}_{it}^{c}+\mu_{t}\cdot\tilde{p}_{Dit}^{c}\cdot\lambda^{c}c_{it}\right\}\right|_{\tilde{x}_{it}^{c}|_{(I.34)}}=0,$$

$$\frac{\partial\left\{-\dfrac{\lambda^{c}}{N_{t}}\cdot2\tilde{\kappa}\tilde{x}_{it}^{c}+\mu_{t}\cdot\tilde{p}_{Dit}^{c}\cdot\lambda^{c}c_{it}\right\}}{\partial\tilde{x}_{it}^{c}}<0.$$

故当式（I.34）计算所得的 $\tilde{x}_{it}^{c}\big|_{(I.34)}>1$（不在取值范围 $[0,1]$ 内）时，可得

$$\Rightarrow\left\{-\frac{\lambda^c}{N_t}\cdot 2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}\right\}\Bigg|_{\tilde{x}_{it}^c\in[0,1]}>\left\{-\frac{\lambda^c}{N_t}\cdot 2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}\right\}\Bigg|_{\tilde{x}_{it}^c|_{(I.34)}}=0,$$

$$\Rightarrow\mu_{\tilde{x}1}^c-\mu_{\tilde{x}0}^c=\left\{-\frac{\lambda^c}{N_t}\cdot 2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}\right\}\Bigg|_{\tilde{x}_{it}^c\in[0,1]}>0,$$

$$\Rightarrow\quad\mu_{\tilde{x}1}^c>0\quad\longrightarrow\quad\tilde{x}_{it}^{c*}=\max\tilde{x}_{it}^{c*}=1.$$

当式（I.34）计算所得的 $\tilde{x}_{it}^c\big|_{(I.34)}<0$（不在取值范围 $[0,1]$ 内）时，可得

$$\Rightarrow\left\{-\frac{\lambda^c}{N_t}\cdot 2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}\right\}\Bigg|_{\tilde{x}_{it}^c\in[0,1]}<\left\{-\frac{\lambda^c}{N_t}\cdot 2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}\right\}\Bigg|_{\tilde{x}_{it}^c|_{(I.34)}}=0,$$

$$\Rightarrow\mu_{\tilde{x}1}^c-\mu_{\tilde{x}0}^c=\left\{-\frac{\lambda^c}{N_t}\cdot 2\tilde{\kappa}\tilde{x}_{it}^c+\mu_t\cdot\tilde{p}_{Dit}^c\cdot\lambda^c c_{it}\right\}\Bigg|_{\tilde{x}_{it}^c\in[0,1]}<0,$$

$$\Rightarrow\quad\mu_{\tilde{x}0}^c>0\quad\longrightarrow\quad\tilde{x}_{it}^{c*}=\min\tilde{x}_{it}^{c*}=0.$$

I.4.4 市场规模 N_t^* 与劳动力配置 $H_{qt}^*:H_{et}^*$

由式(I.20)，将数据市场出清条件、式(I.10)、式(I.11)、式(I.24)以及 $\left(Y_t/Y_{it}\right)^{\frac{1}{\sigma}}=N_t^{\frac{1}{\sigma-1}}$ 代入可得，

$$\Rightarrow w_{Ht}=\frac{(1-\eta)\cdot\left\{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)+\begin{bmatrix}(1-\tau_Y)\dfrac{\alpha}{1-\alpha}\lambda^c x_{it}^c\cdot N_t^{-1}\\+(1-\tau_Y)(1+s_D^c)\lambda^c\tilde{x}_{it}^c+(1+s_D^f)(1-\lambda^c)\tilde{x}_{it}^f\end{bmatrix}\cdot\dfrac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}}\right\}}{\left[\left(\dfrac{L_{yit}z_L}{H_{yit}z_H}\right)^{\frac{\zeta-1}{\zeta}}+1\right]\cdot\left[1-\dfrac{\eta\alpha(1-\lambda^c)x_{it}^f\cdot N_t^{-1}}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]}\right]}\cdot\frac{Y_{it}N_t^{\frac{1}{\sigma-1}}}{H_{yit}}.\tag{I.35}$$

由产品市场自由进出条件，将式（I.30）、式（I.31）代入可得，

$$\Rightarrow w_{Ht}=\frac{V_{it}}{\chi}\left[1+\frac{\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}{g_N}\right]=\frac{(1-\tau_Y)/\sigma}{\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+\rho+(g_N-n_{\bar{L}})}\cdot\left[\frac{1}{\chi}+\frac{\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}{\chi g_N}\right]\cdot Y_{it}N_t^{\frac{1}{\sigma-1}}.\tag{I.36}$$

联立式（I.35）和式（I.36），并将式（I.28）代入可得

$$\begin{aligned}\Rightarrow w_{Ht}&=\frac{(1-\eta)\cdot\left\{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)+\begin{bmatrix}(1-\tau_Y)\dfrac{\alpha}{1-\alpha}\lambda^c x_{it}^c\cdot N_t^{-1}\\+(1-\tau_Y)(1+s_D^c)\lambda^c\tilde{x}_{it}^c+(1+s_D^f)(1-\lambda^c)\tilde{x}_{it}^f\end{bmatrix}\cdot\dfrac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}}\right\}}{\left[\left(\dfrac{L_{yit}z_L}{H_{yit}z_H}\right)^{\frac{\zeta-1}{\zeta}}+1\right]\cdot\left[1-\dfrac{\eta\alpha(1-\lambda^c)x_{it}^f\cdot N_t^{-1}}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]}\right]}\cdot\frac{Y_{it}N_t^{\frac{1}{\sigma-1}}}{H_{yit}}\\&=\frac{(1-\tau_Y)/\sigma}{\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+\rho+(g_N-n_{\bar{L}})}\cdot\left[\frac{1}{\chi}+\frac{\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}{\chi g_N}\right]\cdot Y_{it}N_t^{\frac{1}{\sigma-1}},\end{aligned}$$

$$\xrightarrow{N_t\to\infty} H_{yit}\cdot\left[\left(\frac{L_{yit}z_L}{H_{yit}z_H}\right)^{\frac{\zeta-1}{\zeta}}+1\right]=\frac{(1-\eta)\cdot\left\{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right)+\left[(1-\tau_Y)(1+s_D^c)\lambda^c\tilde{x}_{it}^c+(1+s_D^f)(1-\lambda^c)\tilde{x}_{it}^f\right]\cdot\frac{\tilde{p}_{Dt}^{mf}}{N_t^{\frac{1}{\sigma-1}-1}}\right\}}{\frac{(1-\tau_Y)/\sigma}{\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+\rho+(g_N-n_{\bar{L}})}\cdot\left[\frac{1}{\chi}+\frac{\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}{\chi g_N}\right]},$$

$$\Rightarrow H_{yit}\left[\left(\frac{L_{yit}z_L}{H_{yit}z_H}\right)^{\frac{\zeta-1}{\zeta}}+1\right]=\frac{g_N\chi(1-\eta)(\sigma-1)\cdot\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\cdot\left[\rho+\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+(g_N-n_{\bar{L}})\right]}{\left\{\begin{matrix}\left[1-(1-\tau_Y)(1+s_D^c)\eta\right]\cdot\lambda^c\tilde{x}_{it}^c\\+\left[1-(1+s_D^f)\eta\right]\cdot(1-\lambda^c)\tilde{x}_{it}^f\end{matrix}\right\}\cdot\left[g_N+\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2\right]}.\quad (\text{I}.37)$$

由式（I. 37）可判断得稳态时 $H_{yit}\left[\left(\frac{L_{yit}z_L}{H_{yit}z_H}\right)^{\frac{\zeta-1}{\zeta}}+1\right]$ 是常数。结合产品种类的动态方程和熟练劳动力市场出清条件可得，

$$\Rightarrow \dot{N}_t=\frac{1}{\chi}H_{et}=\frac{1}{\chi}\left(l_H\bar{L}_t-N_tH_{yit}\right),$$

$$\Rightarrow g_{N_t}=\frac{\dot{N}_t}{N_t}=\frac{1}{\chi}\left(\frac{l_H\bar{L}_t}{N_t}-H_{yit}\right).\quad (\text{I}.38)$$

判断稳态时 g_{N_t} 和 H_{yit} 是常数，故结合 $\bar{L}_t=e^{n_{\bar{L}}\cdot t}\cdot\bar{L}_0$ 和式（I. 38）可得稳态时需满足：

$$\Rightarrow g_N=g_{\bar{L}}=n_{\bar{L}}.\quad (\text{I}.39)$$

联立式（I. 38）与式（I. 39），可得，

$$\Rightarrow N_t=\frac{l_H\bar{L}_t}{n_{\bar{L}}\chi+H_{yit}}=\frac{l_H\bar{L}_0}{n_{\bar{L}}\chi+H_{yit}}\cdot e^{n_{\bar{L}}\cdot t},\quad (\text{I}.40)$$

$$\Rightarrow\frac{H_{et}}{H_{yt}}=\frac{l_H\bar{L}_t-N_tH_{yit}}{N_tH_{yit}}=\frac{l_H\bar{L}_t-\frac{l_H\bar{L}_t}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}}{\frac{l_H\bar{L}_t}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}}=\frac{n_{\bar{L}}\chi}{H_{yit}}.$$

也即：

$$H_{et}=\frac{n_{\bar{L}}\chi}{n_{\bar{L}}\chi+H_{yit}}\cdot H_t=\frac{n_{\bar{L}}\chi}{n_{\bar{L}}\chi+H_{yit}}\cdot l_H\bar{L}_t=\frac{n_{\bar{L}}\chi\cdot l_H\bar{L}_0}{n_{\bar{L}}\chi+H_{yit}}\cdot e^{n_{\bar{L}}\cdot t},$$

$$H_{yt}=\frac{H_{yit}}{n_{\bar{L}}\chi+H_{yit}}\cdot H_t=\frac{H_{yit}}{n_{\bar{L}}\chi+H_{yit}}\cdot l_H\bar{L}_t=\frac{H_{yit}\cdot l_H\bar{L}_0}{n_{\bar{L}}\chi+H_{yit}}\cdot e^{n_{\bar{L}}\cdot t}.$$

同时，由非熟练劳动力市场出清条件，将式（I. 40）代入可得，

$$L_{yit}=\frac{(1-l_H)\bar{L}_t}{N_t}=\frac{(1-l_H)\bar{L}_t}{\left(\frac{l_H\bar{L}_t}{n_{\bar{L}}\chi+H_{yit}}\right)}=\frac{1-l_H}{l_H}\cdot\left(n_{\bar{L}}\chi+H_{yit}\right).\quad (\text{I}.41)$$

将式（I. 39）和式（I. 41）代入式（I. 37）可得，

$$\Rightarrow H_{yit}\left\{\left[\frac{(1-l_H)z_L}{l_H z_H}\cdot\left(\frac{n_{\bar{L}}\chi}{H_{yit}}+1\right)\right]^{\frac{\zeta-1}{\zeta}}+1\right\}=\frac{n_{\bar{L}}\chi(1-\eta)(\sigma-1)\cdot\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]}{\left\{\begin{array}{l}\left[1-(1-\tau_Y)(1+s_D^c)\eta\right]\cdot\lambda^c\tilde{x}_{it}^c\\+\left[1-(1+s_D^f)\eta\right]\cdot(1-\lambda^c)\tilde{x}_{it}^f\end{array}\right\}}\cdot\left[\frac{\rho+\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}{n_{\bar{L}}+\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}\right]$$
$$=\frac{n_{\bar{L}}\chi(1-\eta)(\sigma-1)\cdot\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]}{\left\{\begin{array}{l}\left[1-(1-\tau_Y)(1+s_D^c)\eta\right]\cdot\lambda^c\tilde{x}_{it}^c\\+\left[1-(1+s_D^f)\eta\right]\cdot(1-\lambda^c)\tilde{x}_{it}^f\end{array}\right\}}\cdot\left[\frac{\rho-n_{\bar{L}}}{n_{\bar{L}}+\delta(1-\lambda^c)(\tilde{x}_{it}^f)^2}+1\right]. \tag{I. 42}$$

此外，将式（I. 39）代入式（I. 33）、式（I. 34）可得，

$$\Rightarrow\frac{2\delta\tilde{x}_{it}^f/\sigma}{\delta(1-\lambda^c)\cdot(\tilde{x}_{it}^f)^2+\rho}=\frac{(1+s_D^f)\left(1-\frac{1}{\sigma}\right)}{\left[\frac{1}{\eta}-(1-\tau_Y)(1+s_D^c)\right]\cdot\lambda^c\tilde{x}_{it}^c+\left[\frac{1}{\eta}-(1+s_D^f)\right]\cdot(1-\lambda^c)\tilde{x}_{it}^f}, \tag{I. 43}$$

$$\Rightarrow\frac{\eta(\sigma-1)}{\left(\frac{1}{1+s_D^f}-\eta\right)\cdot 2\sigma\tilde{\kappa}\tilde{x}_{it}^c}-\frac{\left[\frac{1}{(1-\tau_Y)(1+s_D^c)}-\eta\right]\cdot\lambda^c\tilde{x}_{it}^c}{\frac{1}{1+s_D^f}-\eta}=\frac{1}{2\sigma\tilde{\kappa}\tilde{x}_{it}^c}-\sqrt{\frac{1}{\left(2\sigma\tilde{\kappa}\tilde{x}_{it}^c\right)^2}-\frac{\rho(1-\lambda^c)(1+s_D^f)^2}{\delta(1-\tau_Y)^2(1+s_D^c)^2}}. \tag{I. 44}$$

由上述结果可知，在微观层面上，消费者和企业数据的每单位共享比例（$\tilde{x}_{it}^c=\tilde{x}_{it}^{c*}=\tilde{x}_i^{c*}$、$\tilde{x}_{it}^f=\tilde{x}_{it}^{f*}=\tilde{x}_i^{f*}$）、每种产品企业的生产规模即投入生产经营的劳动力数量（$L_{yit}=L_{yit}^*=L_{yi}^*$、$H_{yit}=H_{yit}^*=H_{yi}^*$）、熟练劳动力在新产品创新和产品生产经营环节的配置比例（$H_{et}^*:H_{yt}^*$）均是长期稳定的常数。

I. 4. 5 经济总产出 Y_t^* 与社会福利 U^*

由企业生产函数，将劳动力市场出清条件、数据市场出清条件、式（I. 40）、式（I. 41）以及 $Y_t=Y_{it}\cdot N_t^{\frac{\sigma}{\sigma-1}}$ 代入，可计算得经济总产出等于：

$$\Rightarrow Y_{it}=\left\{\alpha\left[D_{it}^c+(1-\lambda^c)x_{it}^f\cdot Y_{it}\right]+(1-\alpha)(\tilde{D}_t^c+\tilde{D}_t^f)\right\}^{\eta}\cdot\left\{\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}\right\}^{1-\eta}$$
$$=Y_{it}^{\eta}\cdot N_t^{\eta}\cdot\left\{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\eta}\cdot\left\{\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}\right\}^{1-\eta},$$

$$\Rightarrow Y_{it}=N_t^{\frac{\eta}{1-\eta}}\cdot\left\{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\frac{\eta}{1-\eta}}\cdot\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}},$$

$$\Rightarrow Y_t=Y_{it}\cdot N_t^{\frac{\sigma}{\sigma-1}}=N_t^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left\{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\frac{\eta}{1-\eta}}\cdot\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}},$$

$$\xrightarrow{N_t\to\infty}Y_t=N_t^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left\{(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\frac{\eta}{1-\eta}}\cdot\left[(L_{yit}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}},$$

$$\Rightarrow Y_t=\left(\frac{l_H\bar{L}_0}{n_{\bar{L}}\chi+H_{yit}}\cdot e^{n_{\bar{L}}\cdot t}\right)^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left\{(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\frac{\eta}{1-\eta}}\cdot\left\{\left[\frac{(1-l_H)z_L}{l_H}\cdot\left(n_{\bar{L}}\chi+H_{yit}\right)\right]^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}}.$$

因此，经济总量层面与人均层面的增长率等于：

$$g_Y=\frac{1}{\sigma-1}\cdot n_{\bar{L}}+\frac{1}{1-\eta}\cdot n_{\bar{L}}=\left(\frac{1}{\sigma-1}+\frac{1}{1-\eta}\right)\cdot n_{\bar{L}},$$

$$g_c=g_{Y/\bar{L}}=g_Y-n_{\bar{L}}=\left(\frac{1}{\sigma-1}+\frac{1}{1-\eta}\right)\cdot n_{\bar{L}}-n_{\bar{L}}=\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)\cdot n_{\bar{L}}. \tag{I.45}$$

由家庭效用函数，将式（I.45）和 $c_t=c_0\cdot e^{g_c\cdot t}=\frac{Y_0}{\bar{L}_0}\cdot e^{g_c\cdot t}$ 代入，可计算得社会福利水平为：

$$U=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})t}\cdot\left\{\ln c_t-\frac{\lambda^c}{N_t^2}\cdot\kappa\cdot\int_0^{N_t}(x_{it}^c)^2di-\frac{\lambda^c}{N_t}\cdot\tilde{\kappa}\cdot\int_0^{N_t}(\tilde{x}_{it}^c)^2di\right\}dt,$$

$$\xrightarrow{\text{对称性}}U=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\left\{\ln\left(\frac{Y_0}{\bar{L}_0}\right)+\left(g_Y-n_{\bar{L}}\right)\cdot t-\frac{\lambda^c}{N_t}\kappa(x_{it}^c)^2-\lambda^c\tilde{\kappa}(\tilde{x}_{it}^c)^2\right\}dt,$$

$$\xrightarrow{N_t\to\infty}U=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)+\left(g_Y-n_{\bar{L}}\right)\cdot t-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2\right]dt$$
$$=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2\right]dt+L_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\left[\left(g_Y-n_{\bar{L}}\right)\cdot t\right]dt$$
$$=\left\{-e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2\right]-e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\left(g_Y-n_{\bar{L}}\right)\cdot t\right]-e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\frac{\bar{L}_0}{(\rho-n_{\bar{L}})^2}\cdot\left(g_Y-n_{\bar{L}}\right)\right\}\Bigg|_{t=0}^{\infty},$$

$$\Rightarrow U=\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2+\frac{1}{\rho-n_{\bar{L}}}\cdot\left(g_Y-n_{\bar{L}}\right)\right]$$
$$=\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2+\frac{1}{\rho-n_{\bar{L}}}\cdot\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)\cdot n_{\bar{L}}\right].$$

I.5 数据交易的政府补贴

已知政府收支平衡条件为：

$$s_D^c\cdot\tilde{p}_{Dt}^{mc}\cdot\int_0^{N_t}\tilde{D}_t^c di+s_D^f\cdot\tilde{p}_{Dt}^{mf}\cdot\int_0^{N_t}\tilde{D}_t^f di=\int_0^{N_t}\tau_Y\cdot\left[p_{Dit}^c\cdot\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c+\left(Y_t/Y_{it}\right)^{1/\sigma}\right]\cdot Y_{it}di,$$

$$\xrightarrow{\text{对称性}}s_D^c\cdot\tilde{p}_{Dt}^{mc}\cdot N_t\tilde{D}_t^c+s_D^f\cdot\tilde{p}_{Dt}^{mf}\cdot N_t\tilde{D}_t^f=N_t\cdot\tau_Y\cdot\left[p_{Dit}^c\cdot\lambda^c x_{it}^c+\tilde{p}_{Dit}^c\cdot\lambda^c\tilde{x}_{it}^c+\left(Y_t/Y_{it}\right)^{1/\sigma}\right]\cdot Y_{it}.$$

将数据市场出清条件、式（I.10）、式（I.24）、式（I.28）以及 $\left(Y_t/Y_{it}\right)^{\frac{1}{\sigma}}=N_t^{\frac{1}{\sigma-1}}$ 代入政府收支平衡条件，可得：

$$
\begin{aligned}
&\Rightarrow \left[s_D^c \cdot \lambda^c \tilde{x}_{it}^c + s_D^f \cdot (1-\lambda^c)\tilde{x}_{it}^f\right] \cdot \tilde{p}_{Dt}^{mf} \cdot Y_{it} \cdot N_t^{\,2} \\
&\quad = \tau_Y \cdot \left\{\left[\frac{\alpha}{1-\alpha} \cdot \lambda^c x_{it}^c \cdot N_t^{-1} + (1+s_D^c)\lambda^c \tilde{x}_{it}^c\right] \cdot \tilde{p}_{Dt}^{mf} + N_t^{\frac{1}{\sigma-1}-1}\right\} \cdot Y_{it} \cdot N_t^{\,2},
\end{aligned}
$$

$$
\Rightarrow \tau_Y = \frac{s_D^c \cdot \lambda^c \tilde{x}_{it}^c + s_D^f \cdot (1-\lambda^c)\tilde{x}_{it}^f}{\left[\frac{\alpha}{1-\alpha} \cdot \lambda^c x_{it}^c \cdot N_t^{-1} + (1+s_D^c)\lambda^c \tilde{x}_{it}^c\right] + \frac{N_t^{\frac{1}{\sigma-1}-1}}{\tilde{p}_{Dt}^{mf}}},
$$

$$
\xrightarrow{N_t \to \infty} \tau_Y = \frac{s_D^c \cdot \lambda^c \tilde{x}_{it}^c + s_D^f \cdot (1-\lambda^c)\tilde{x}_{it}^f}{(1+s_D^c)\lambda^c \tilde{x}_{it}^c + \frac{N_t^{\frac{1}{\sigma-1}-1}}{\tilde{p}_{Dt}^{mf}}},
$$

$$
\Rightarrow \tau_Y = \frac{(1-\tau_Y)\left(1-\frac{1}{\sigma}\right) \cdot \left[s_D^c \cdot \lambda^c \tilde{x}_{it}^c + s_D^f \cdot (1-\lambda^c)\tilde{x}_{it}^f\right]}{\left[\frac{1}{\eta} - (1-\tau_Y)(1+s_D^c)\frac{1}{\sigma}\right] \cdot \lambda^c \tilde{x}_{it}^c + \left[\frac{1}{\eta} - (1+s_D^f)\right] \cdot (1-\lambda^c)\tilde{x}_{it}^f}. \tag{I.46}
$$

附录 II 社会计划者均衡的推导求解过程

II.1 社会计划者

结合产品种类的动态方程和熟练劳动力市场出清条件可得，

$$\Rightarrow \dot{N}_t = \frac{1}{\chi} H_{et} = \frac{1}{\chi}\left(l_H \bar{L}_t - N_t H_{yit}\right),$$

$$\Rightarrow g_{N_t} = \frac{\dot{N}_t}{N_t} = \frac{1}{\chi}\cdot\frac{l_H \bar{L}_t - N_t H_{yit}}{N_t} = \frac{1}{\chi}\left(\frac{l_H \bar{L}_t}{N_t} - H_{yit}\right). \qquad \text{(II.1)}$$

判断稳态时 g_N 和 H_{yit} 是常数，故结合 $\bar{L}_t = e^{n_{\bar{L}}\cdot t}\cdot \bar{L}_0$ 和式（II.1）可得稳态时需满足：

$$\Rightarrow g_N = g_{\bar{L}} = n_{\bar{L}}.$$

代入式（II.1）可得，

$$\Rightarrow N_t = \frac{l_H \bar{L}_t}{n_{\bar{L}}\chi + H_{yit}} = \frac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}. \qquad \text{(II.2)}$$

因此，改写社会计划者的最优化问题：

$$\max U = \bar{L}_0 \cdot \int_0^{\infty} e^{-(\rho - n_{\bar{L}})t} \cdot \left\{\begin{array}{l} \ln \dfrac{N_t^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}} \cdot \left\{\begin{array}{l}\alpha\left[\lambda^c x_{it}^c + (1-\lambda^c) x_{it}^f\right]\cdot N_t^{-1} \\ +(1-\alpha)\left[\lambda^c \tilde{x}_{it}^c + (1-\lambda^c)\tilde{x}_{it}^f\right]\end{array}\right\}^{\frac{\eta}{1-\eta}} \cdot \left\{\left[(1-l_H)\bar{L}_t z_L\right]^{\frac{\zeta-1}{\zeta}} + (H_{yt} z_H)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}}}{\bar{L}_t} \\ -\dfrac{\lambda^c}{N_t}\cdot \kappa \cdot (x_{it}^c)^2 - \lambda^c \tilde{\kappa}\cdot (\tilde{x}_{it}^c)^2 \end{array}\right\} dt$$

$$= \bar{L}_0 \cdot \int_0^{\infty} e^{-(\rho - n_{\bar{L}})t} \cdot \left\{\begin{array}{l} \ln\left\{\left(\dfrac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}\right)^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}} \cdot \left\{\begin{array}{l}\alpha\left[\lambda^c x_{it}^c + (1-\lambda^c) x_{it}^f\right]\cdot \left(\dfrac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}\right)^{-1} \\ +(1-\alpha)\left[\lambda^c \tilde{x}_{it}^c + (1-\lambda^c)\tilde{x}_{it}^f\right]\end{array}\right\}^{\frac{\eta}{1-\eta}} \cdot \left\{\left[(1-l_H) z_L\right]^{\frac{\zeta-1}{\zeta}} + \left(\dfrac{l_H}{n_{\bar{L}}\chi + H_{yit}}\cdot H_{yit} z_H\right)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}}\right\} \\ -\dfrac{\lambda^c}{\left(\dfrac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}\right)}\cdot \kappa \cdot (x_{it}^c)^2 - \lambda^c \tilde{\kappa}\cdot (\tilde{x}_{it}^c)^2 \end{array}\right\} dt,$$

$$s.t.\quad 0 \le x_{it}^c \le 1,\quad 0 \le x_{it}^f \le 1,\quad 0 \le \tilde{x}_{it}^c \le 1,\quad 0 \le \tilde{x}_{it}^f \le 1.$$

结合社会计划者的最优化问题，构建拉格朗日（Lagrangean）方程：

$$\mathbf{L} = \left\{\begin{array}{l} \ln\left\{\left(\dfrac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}\right)^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}} \cdot \left\{\begin{array}{l}\alpha\left[\lambda^c x_{it}^c + (1-\lambda^c) x_{it}^f\right]\cdot \left(\dfrac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}\right)^{-1} \\ +(1-\alpha)\left[\lambda^c \tilde{x}_{it}^c + (1-\lambda^c)\tilde{x}_{it}^f\right]\end{array}\right\}^{\frac{\eta}{1-\eta}} \cdot \left\{\left[(1-l_H) z_L\right]^{\frac{\zeta-1}{\zeta}} + \left(\dfrac{l_H}{n_{\bar{L}}\chi + H_{yit}}\cdot H_{yit} z_H\right)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}}\right\} \\ -\dfrac{\lambda^c}{\left(\dfrac{l_H \bar{L}_0 \cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi + H_{yit}}\right)}\cdot \kappa \cdot (x_{it}^c)^2 - \lambda^c \tilde{\kappa}\cdot (\tilde{x}_{it}^c)^2 + \mu_{x0}^c \cdot x_{it}^c + \mu_{x1}^c \cdot (1 - x_{it}^c) + \mu_{\tilde{x}0}^c \cdot \tilde{x}_{it}^c + \mu_{\tilde{x}1}^c \cdot (1 - \tilde{x}_{it}^c) \\ +\mu_{x0}^f \cdot x_{it}^f + \mu_{x1}^f \cdot (1 - x_{it}^f) + \mu_{\tilde{x}0}^f \cdot \tilde{x}_{it}^f + \mu_{\tilde{x}1}^f \cdot (1 - \tilde{x}_{it}^f) \end{array}\right\}.$$

可推导得一阶条件包括：

$$\frac{\partial \mathbf{L}}{\partial x_{it}^c} = \frac{\frac{\eta}{1-\eta}\cdot\alpha\lambda^c\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} - \frac{2\lambda^c\kappa x_{it}^c}{\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)} + \mu_{x0}^c - \mu_{x1}^c = 0, \tag{II.3}$$

$$\frac{\partial \mathbf{L}}{\partial \tilde{x}_{it}^c} = \frac{\frac{\eta}{1-\eta}\cdot(1-\alpha)\lambda^c}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} - 2\lambda^c\kappa\tilde{x}_{it}^c + \mu_{\tilde{x}0}^c - \mu_{\tilde{x}1}^c = 0, \tag{II.4}$$

$$\frac{\partial \mathbf{L}}{\partial x_{it}^f} = \frac{\frac{\eta}{1-\eta}\cdot\alpha(1-\lambda^c)\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} + \mu_{x0}^f - \mu_{x1}^f = 0, \tag{II.5}$$

$$\frac{\partial \mathbf{L}}{\partial \tilde{x}_{it}^f} = \frac{\frac{\eta}{1-\eta}\cdot(1-\alpha)(1-\lambda^c)}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} + \mu_{\tilde{x}0}^f - \mu_{\tilde{x}1}^f = 0, \tag{II.6}$$

$$\frac{\partial \mathbf{L}}{\partial H_{yit}} = \left\{\begin{array}{l} -\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)\cdot\frac{1}{n_{\bar{L}}\chi+H_{yit}}+\frac{\eta}{1-\eta}\frac{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}\right)^{-1}}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} \\ +\frac{\left(\frac{l_H}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}z_H\right)^{\frac{\zeta-1}{\zeta}-1}\cdot\left[-\frac{l_H}{(n_{\bar{L}}\chi+H_{yit})^2}\cdot H_{yit}z_H+\frac{l_H}{n_{\bar{L}}\chi+H_{yit}}\cdot z_H\right]}{\left[(1-l_H)z_L\right]^{\frac{\zeta-1}{\zeta}}+\left(\frac{l_H z_H}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}\right)^{\frac{\zeta-1}{\zeta}}} - \frac{\lambda^c}{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}\cdot\kappa\cdot(x_{it}^c)^2 \end{array}\right\} = 0. \tag{II.7}$$

II.2 数据的共享使用比例 $(\tilde{x}_{it}^{c,sp},\tilde{x}_{it}^{f,sp})$

由一阶条件式（II.5），易得：

$$\mu_{x1}^f - \mu_{x0}^f = \frac{\frac{\eta}{1-\eta}\cdot\alpha(1-\lambda^c)\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} > 0,$$

$$\Rightarrow \quad \mu_{x1}^f > 0 \quad \Rightarrow \quad x_{it}^{f,sp} = \max x_{it}^f = 1. \tag{II.8}$$

由一阶条件式（II.6），易得：

$$\mu_{\tilde{x}1}^f - \mu_{\tilde{x}0}^f = \frac{\frac{\eta}{1-\eta}\cdot(1-\alpha)(1-\lambda^c)}{\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]} > 0,$$

$$\Rightarrow \quad \mu_{\tilde{x}1}^f > 0 \quad \Rightarrow \quad \tilde{x}_{it}^{f,sp} = \max \tilde{x}_{it}^f = 1. \tag{II.9}$$

联立一阶条件式（II.3）和式（II.4），并将式（II.8）和式（II.9）代入可得：

$$x_{it}^{c}=\left\{\frac{\alpha}{1-\alpha}\cdot\left[\frac{\tilde{\kappa}}{\kappa}\cdot\tilde{x}_{it}^{c}+\frac{\mu_{\tilde{x}1}^{c}-\mu_{\tilde{x}0}^{c}}{2\lambda^{c}\kappa}\right]-\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)\cdot\frac{\mu_{x1}^{c}-\mu_{x0}^{c}}{2\lambda^{c}\kappa}\right\}^{\frac{1}{\phi-1}}, \tag{II.10}$$

$$\left\{\frac{\alpha}{1-\alpha}\left[\lambda^{c}x_{it}^{c}+(1-\lambda^{c})\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+\left[\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\right]\right\}\cdot\left[2\tilde{\kappa}\tilde{x}_{it}^{c}+\frac{\mu_{\tilde{x}1}^{c}-\mu_{\tilde{x}0}^{c}}{\lambda^{c}}\right]=\frac{\eta}{1-\eta},$$

$$\xrightarrow{N_t=\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\to\infty}\left[\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\right]\cdot\left[2\tilde{\kappa}\tilde{x}_{it}^{c}+\frac{\mu_{\tilde{x}1}^{c}-\mu_{\tilde{x}0}^{c}}{\lambda^{c}}\right]=\frac{\eta}{1-\eta}. \tag{II.11}$$

因此，$\tilde{x}_{it}^{c}=\tilde{x}_{it}^{c,sp}$ 和 $\tilde{x}_{it}^{f}=\tilde{x}_{it}^{f,sp}$ 将由式（II.11）与式（II.9）的等式结果计算得到。同时，$x_{it}^{c}=x_{it}^{c,sp}$ 和 $x_{it}^{f}=x_{it}^{f,sp}$ 将由式（II.10）与式（II.8）计算得到。

首先，考虑社会计划者最优 $\tilde{x}_{it}^{c}=\tilde{x}_{it}^{c,sp}$ 是内点解的情况，也即满足条件：

$$\tilde{x}_{it}^{c,sp}\in(0,1),\quad \mu_{\tilde{x}0}^{c}=\mu_{\tilde{x}1}^{c}=0.$$

代入式（II.11），可得简化为：

$$\Rightarrow\left[\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\right]\cdot 2\tilde{\kappa}\tilde{x}_{it}^{c}=\frac{\eta}{1-\eta},$$

$$\Rightarrow\tilde{x}_{it}^{c}=\frac{\sqrt{(1-\lambda^{c})^{2}+\frac{2\eta\lambda^{c}}{(1-\eta)\tilde{\kappa}}}-(1-\lambda^{c})}{2\lambda^{c}}. \tag{II.12}$$

其次，考虑社会计划者最优 $\tilde{x}_{it}^{c}=\tilde{x}_{it}^{c,sp}$ 是角点解的情况。类似附录 I 分析易得，基于式（II.12）计算所得的 $\tilde{x}_{it}^{c}\big|_{(II.12)}$ 不在取值范围 $[0,1]$ 内时，

$$\tilde{x}_{it}^{c}\big|_{(II.12)}>1\qquad\Rightarrow\qquad\tilde{x}_{it}^{c,sp}=1,$$
$$\tilde{x}_{it}^{c}\big|_{(II.12)}<0\qquad\Rightarrow\qquad\tilde{x}_{it}^{c,sp}=0.$$

II.3 市场规模 N_t^{sp} 与劳动力配置 $H_{yt}^{sp}:H_{et}^{sp}$

由一阶条件式（II.7）可得，

$$\Rightarrow\left\{\begin{array}{l}\dfrac{\eta}{1-\eta}\dfrac{\alpha\left[\lambda^{c}x_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}}{\alpha\left[\lambda^{c}x_{it}^{c}+(1-\lambda^{c})x_{it}^{f}\right]\cdot\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)^{-1}+(1-\alpha)\left[\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\tilde{x}_{it}^{f}\right]}-\dfrac{\lambda^{c}}{\left(\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\right)}\cdot\kappa\cdot(x_{it}^{c})^{2}\\[2em]+\dfrac{\left(\frac{l_H}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}z_H\right)^{\frac{\zeta-1}{\zeta}-1}\cdot\left[-\frac{l_H}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}z_H+l_H\cdot z_H\right]}{\left[(1-l_H)z_L\right]^{\frac{\zeta-1}{\zeta}}+\left(\frac{l_Hz_H}{n_{\bar{L}}\chi+H_{yit}}\cdot H_{yit}\right)^{\frac{\zeta-1}{\zeta}}}\end{array}\right\}=\frac{1}{\sigma-1}+\frac{\eta}{1-\eta},$$

$$\xrightarrow{N_t=\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{n_{\bar{L}}\chi+H_{yit}}\to\infty}H_{yit}\cdot\left\{\left[\frac{(1-l_H)z_L}{l_Hz_H}\cdot\left(\frac{n_{\bar{L}}\chi}{H_{yit}}+1\right)\right]^{\frac{\zeta-1}{\zeta}}+1\right\}=\frac{n_{\bar{L}}\chi}{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}}. \tag{II.13}$$

同时，结合式（II.2）以及劳动力市场出清条件可得：

$$\Rightarrow \frac{H_{et}}{H_{yt}}=\frac{l_H\bar{L}_t-H_{yt}}{H_{yt}}=\frac{l_H\bar{L}_t-H_{yit}\cdot\dfrac{l_H\bar{L}_t}{n_{\bar{L}}\chi+H_{yit}}}{H_{yit}\cdot\dfrac{l_H\bar{L}_t}{n_{\bar{L}}\chi+H_{yit}}}=\frac{n_{\bar{L}}\chi}{H_{yit}},$$

$$\Rightarrow L_{yit}=\frac{(1-l_H)\bar{L}_t}{N_t}=\frac{1-l_H}{l_H}\cdot\left(n_{\bar{L}}\chi+H_{yit}\right). \tag{II.14}$$

II.4 经济总产出 Y_t^{sp} 与社会福利 U^{sp}

接下来，由企业生产函数，将劳动力市场出清条件、式（II.2）、式（II.14）代入，可计算得经济总产出等于：

$$\Rightarrow Y_t=N_t^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\begin{Bmatrix}\alpha\left[\lambda^c x_{it}^c+(1-\lambda^c)x_{it}^f\right]\cdot N_t^{-1}\\+(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\end{Bmatrix}^{\frac{\eta}{1-\eta}}\cdot\left\{\left[L_{yit}z_L\right]^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}},$$

$$\xrightarrow{N_t\to\infty}Y_t=N_t^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left\{(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\frac{\eta}{1-\eta}}\cdot\left\{\left[L_{yit}z_L\right]^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}},$$

$$\Rightarrow Y_t=\left(\frac{l_H\bar{L}_0}{n_L\chi+H_{yit}}\cdot e^{n_{\bar{L}}\cdot t}\right)^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left\{(1-\alpha)\left[\lambda^c\tilde{x}_{it}^c+(1-\lambda^c)\tilde{x}_{it}^f\right]\right\}^{\frac{\eta}{1-\eta}}\cdot\left\{\left[\frac{(1-l_H)z_L}{l_H}\cdot\left(n_{\bar{L}}\chi+H_{yit}\right)\right]^{\frac{\zeta-1}{\zeta}}+(H_{yit}z_H)^{\frac{\zeta-1}{\zeta}}\right\}^{\frac{\zeta}{\zeta-1}}.$$

因此，经济总量层面与人均层面的增长率等于：

$$g_Y=\left(\frac{1}{\sigma-1}+\frac{1}{1-\eta}\right)n_{\bar{L}},$$

$$g_{Y/\bar{L}}=g_Y-n_{\bar{L}}=\left(\frac{1}{\sigma-1}+\frac{1}{1-\eta}\right)n_{\bar{L}}-n_{\bar{L}}=\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)n_{\bar{L}}. \tag{II.15}$$

由家庭效用函数，将式（II.15）和 $\frac{Y_t}{\bar{L}_t}=\frac{Y_0}{\bar{L}_0}\cdot e^{g_{Y/\bar{L}}\cdot t}$ 代入，可计算得社会福利水平为：

$$\begin{aligned}U&=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})t}\cdot\left\{\ln\left(\frac{Y_t}{\bar{L}_t}\right)-\frac{\lambda^c}{N_t}\cdot\kappa\cdot(x_{it}^c)^2-\lambda^c\tilde{\kappa}\cdot(\tilde{x}_{it}^c)^2\right\}dt\\&=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})t}\cdot\left\{\ln\left(\frac{Y_0}{\bar{L}_0}\right)+g_{Y/\bar{L}}\cdot t-\frac{\lambda^c}{N_t}\cdot\kappa\cdot(x_{it}^c)^2-\lambda^c\tilde{\kappa}\cdot(\tilde{x}_{it}^c)^2\right\}dt,\end{aligned}$$

$$\begin{aligned}\xrightarrow{N_t\to\infty}U&=\bar{L}_0\cdot\int_0^\infty e^{-(\rho-n_{\bar{L}})\cdot t}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)+\left(g_Y-n_{\bar{L}}\right)\cdot t-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2\right]dt\\&=\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2+\frac{1}{\rho-n_{\bar{L}}}\cdot\left(g_Y-n_{\bar{L}}\right)\right]\\&=\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^c)^2+\frac{1}{\rho-n_{\bar{L}}}\cdot\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)\cdot n_{\bar{L}}\right].\end{aligned}$$

附录 III 经济均衡结果理论分析的推导求解过程

III.1 分散经济均衡结果理论分析

III.1.1 分散经济的数据共享均衡结果

由分散经济的数据共享均衡结果

$2\tilde{\kappa}\cdot\tilde{x}_i^{c*}=\dfrac{2\delta\cdot\tilde{x}_i^{f*}}{\sigma\cdot\left[\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2+\rho\right]}=\dfrac{\eta(\sigma-1)}{(1-\eta)\sigma\cdot\left[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]}$，推导可得：

$$\Rightarrow\begin{cases}\tilde{x}_i^{c*}\big|_{\lambda^c=0}=\dfrac{\sqrt{\delta}}{\tilde{\kappa}\cdot\sigma\sqrt{\rho}}\cdot\sqrt{\dfrac{\eta(\sigma-1)}{2(1-\eta)}\left[1-\dfrac{\eta(\sigma-1)}{2(1-\eta)}\right]},\\ \tilde{x}_i^{f*}\big|_{\lambda^c=0}=\sqrt{\dfrac{\rho\eta(\sigma-1)}{\delta\cdot\left[2(1-\eta)-\eta(\sigma-1)\right]}},\\ \tilde{x}_i^{c*}\big|_{\lambda^c=1}=\sqrt{\dfrac{\eta(\sigma-1)}{\tilde{\kappa}\sigma\cdot 2(1-\eta)}},\\ \tilde{x}_i^{f*}\big|_{\lambda^c=1}=\dfrac{\sqrt{\tilde{\kappa}}\cdot\rho\sqrt{\sigma}}{\delta}\cdot\sqrt{\dfrac{\eta(\sigma-1)}{2(1-\eta)}}.\end{cases}\tag{III.1}$$

其中式（III.1）表明算术平方根存在解的条件是 $2(1-\eta)\geq\eta(\sigma-1)$，也即 $\eta\leq\dfrac{2}{\sigma+1}$。

由 $2\tilde{\kappa}\cdot\tilde{x}_i^{c*}=\dfrac{\eta(\sigma-1)}{(1-\eta)\sigma\cdot\left[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]}$ 可得：

$$\Rightarrow\tilde{x}_i^{f*}=\frac{1}{1-\lambda^c}\left[\frac{\eta(\sigma-1)}{(1-\eta)\sigma\cdot 2\tilde{\kappa}\tilde{x}_i^{c*}}-\lambda^c\tilde{x}_i^{c*}\right].\tag{III.2}$$

由 $2\tilde{\kappa}\cdot\tilde{x}_i^{c*}=\dfrac{2\delta\cdot\tilde{x}_i^{f*}}{\sigma\cdot\left[\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2+\rho\right]}$，将式（III.2）代入可得：

$$\Rightarrow\left[\frac{\eta(\sigma-1)}{(1-\eta)\cdot 2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-\lambda^c\tilde{x}_i^{c*}\right]^2-\frac{1}{\sigma\tilde{\kappa}\tilde{x}_i^{c*}}\cdot\left[\frac{\eta(\sigma-1)}{(1-\eta)\cdot 2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-\lambda^c\tilde{x}_i^{c*}\right]+\frac{\rho(1-\lambda^c)}{\delta}=0,$$

$$\Rightarrow\frac{\eta(\sigma-1)}{(1-\eta)\cdot 2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-\lambda^c\tilde{x}_i^{c*}=\frac{1}{2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-\sqrt{\frac{1}{\left(2\sigma\tilde{\kappa}\tilde{x}_i^{c*}\right)^2}-\frac{\rho(1-\lambda^c)}{\delta}}.$$

等式两边关于 λ^c 求全导，可得对消费者数据的共享情况而言：

$$\frac{\partial\tilde{x}_i^{c*}}{\partial\lambda^c}=\frac{\dfrac{\rho}{2\delta}\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}-1}{\dfrac{\rho}{2\delta}\cdot 2(1-\lambda^c)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2\lambda^c}\cdot\tilde{x}_i^{c*},\tag{III.3}$$

$$\Rightarrow\frac{\partial\lambda^c\tilde{x}_i^{c*}}{\partial\lambda^c}=\tilde{x}_i^{c*}+\lambda^c\cdot\frac{\partial\tilde{x}_i^{c*}}{\partial\lambda^c}=\frac{\dfrac{\rho}{2\delta}\cdot\left[(1-\lambda^c)+1\right]\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+\lambda^c}{\dfrac{\rho}{2\delta}\cdot 2(1-\lambda^c)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2\lambda^c}\cdot\tilde{x}_i^{c*}\geq 0,\tag{III.4}$$

$$\Rightarrow \frac{\partial \lambda^c (\tilde{x}_i^{c*})^2}{\partial \lambda^c} = (\tilde{x}_i^{c*})^2 + 2\lambda^c \tilde{x}_i^{c*} \cdot \frac{\partial \tilde{x}_i^{c*}}{\partial \lambda^c} = \frac{\frac{\rho}{2\delta} \cdot 2 \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] + \lambda^c (\tilde{x}_i^{c*})^2 \right\}^{-1} \cdot (\tilde{x}_i^{c*})^2}{\frac{\rho}{2\delta} \cdot 2(1-\lambda^c) \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] + \lambda^c (\tilde{x}_i^{c*})^2 \right\}^{-1} + 2\lambda^c} \geq 0. \quad \text{(III.5)}$$

也即，式(III.4)和式(III.5)可证得 $\lambda^c \tilde{x}_i^{c*}$ 和 $\lambda^c (\tilde{x}_i^{c*})^2$ 关于 λ^c 递增。

由式(III.3)，代入式(III.5)中 $\lambda^c (\tilde{x}_i^{c*})^2$ 关于 λ^c 递增的关系，可得 $sign\left(\frac{\partial \tilde{x}_i^{c*}}{\partial \lambda^c} \right) = sign\left\{ \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] + \lambda^c (\tilde{x}_i^{c*})^2 \right\}^{-1} - 1 \right\}$，且该式关于 λ^c 递减。继续将式(III.1)代入可得：

$$\left\{ \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{(1-\eta)} \right] + \lambda^c (\tilde{x}_i^{c*})^2 \right\}^{-1} - 1 \right\} \Bigg|_{\lambda^c=0} = \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] \right\}^{-1} - 1,$$

$$\left\{ \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] + \lambda^c (\tilde{x}_i^{c*})^2 \right\}^{-1} - 1 \right\} \Bigg|_{\lambda^c=1} = \frac{\tilde{\kappa}}{\delta} \cdot \rho\sigma - 1,$$

$$\Rightarrow \left\{ \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] + \lambda^c (\tilde{x}_i^{c*})^2 \right\}^{-1} - 1 \right\} \in \left[\frac{\tilde{\kappa}}{\delta} \cdot \rho\sigma - 1, \quad \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] \right\}^{-1} - 1 \right].$$

$$\Rightarrow \frac{\partial \tilde{x}_i^{c*}}{\partial \lambda^c} = \begin{cases} \leq 0,\ \tilde{x}_i^{c*}\text{关于}\lambda^c\text{边际递减}, & if\ \ \frac{\rho}{2\delta} \cdot \left\{ \frac{1}{2\sigma\tilde{\kappa}} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] \right\}^{-1} - 1 \leq 0,\text{也即}\frac{\tilde{\kappa}}{\delta} \leq \frac{1}{\rho\sigma} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right], \\ \text{先}>0,\text{后}<0,\ \tilde{x}_i^{c*}\text{关于}\lambda^c\text{先递增后递减}, & if\ \ \frac{1}{\rho\sigma} \left[1 - \frac{\eta(\sigma-1)}{1-\eta} \right] < \frac{\tilde{\kappa}}{\delta} < \frac{1}{\rho\sigma}, \\ \geq 0,\ \tilde{x}_i^{c*}\text{关于}\lambda^c\text{边际递增}, & if\ \ \frac{\tilde{\kappa}}{\delta} \cdot \rho\sigma - 1 \geq 0,\text{也即}\frac{\tilde{\kappa}}{\delta} \geq \frac{1}{\rho\sigma}. \end{cases} \quad \text{(III.6)}$$

由 $2\tilde{\kappa} \cdot \tilde{x}_i^{c*} = \frac{\eta(\sigma-1)}{(1-\eta)\sigma \cdot \left[\lambda^c \tilde{x}_i^{c*} + (1-\lambda^c) \tilde{x}_i^{f*} \right]}$，可得整体数据平均共享程度为 $\tilde{x}_i^* = \lambda^c \tilde{x}_i^{c*} + (1-\lambda^c) \tilde{x}_i^{f*} = \frac{\eta(\sigma-1)}{(1-\eta)\sigma \cdot 2\tilde{\kappa} \cdot \tilde{x}_i^{c*}}$，因此：

$$\Rightarrow \frac{\partial \tilde{x}_i^*}{\partial \lambda^c} = \frac{\partial \left[\lambda^c \tilde{x}_i^{c*} + (1-\lambda^c) \tilde{x}_i^{f*} \right]}{\partial \lambda^c} = -\frac{\eta(\sigma-1)}{(1-\eta)\sigma \cdot 2\tilde{\kappa} \cdot (\tilde{x}_i^{c*})^2} \cdot \frac{\partial \tilde{x}_i^{c*}}{\partial \lambda^c}, \quad \text{(III.7)}$$

$$\Rightarrow sign\left\{ \frac{\partial \tilde{x}_i^*}{\partial \lambda^c} \right\} = sign\left(-\frac{\partial \tilde{x}_i^{c*}}{\partial \lambda^c} \right),$$

$$\Rightarrow \frac{\partial \tilde{x}_i^*}{\partial \lambda^c}=\begin{cases}\geq 0,\ \tilde{x}_i^*\text{关于}\lambda^c\text{边际递增}, & if\ \dfrac{\tilde{\kappa}}{\delta}\leq \dfrac{1}{\rho\sigma}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right],\\ \text{先}<0,\text{后}>0,\ \tilde{x}_i^*\text{关于}\lambda^c\text{先递减后递增}, & if\ \dfrac{1}{\rho\sigma}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]<\dfrac{\tilde{\kappa}}{\delta}<\dfrac{1}{\rho\sigma},\\ \leq 0,\ \tilde{x}_i^*\text{关于}\lambda^c\text{边际递减}, & if\ \dfrac{\tilde{\kappa}}{\delta}\geq \dfrac{1}{\rho\sigma}.\end{cases} \quad \text{(III.8)}$$

也即，式（III.6）和式（III.8）可证得 $\tilde{x}_i^{c*}$ 和 $\tilde{x}_i^*$ 关于 λ^c 的变动关系式存在三种不同的情况，分别对应着两类数据共享的成本强度参数的三种不同数值关系。同时，第一种情况时 $\frac{\tilde{\kappa}}{\delta}\leq \frac{1}{\rho\sigma}\left[1-\frac{\eta(\sigma-1)}{1-\eta}\right]$ 的存在条件是 $1-\frac{\eta(\sigma-1)}{1-\eta}>0$，也即 $\eta<\frac{1}{\sigma}$。

由 $2\tilde{\kappa}\cdot \tilde{x}_i^{c*}=\dfrac{2\delta\cdot \tilde{x}_i^{f*}}{\sigma\cdot\left[\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2+\rho\right]}=\dfrac{\eta(\sigma-1)}{(1-\eta)\sigma\cdot\left[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]}$ 又可得：

$$\frac{\tilde{x}_i^{c*}}{\tilde{x}_i^{f*}}=\frac{\delta}{\tilde{\kappa}}\cdot\frac{1}{\sigma\rho}-\frac{\delta}{\rho}(1-\lambda^c)\tilde{x}_i^{c*}\tilde{x}_i^{f*}=\frac{\delta}{\tilde{\kappa}}\cdot\frac{1}{\sigma\rho}\cdot\left[1-\frac{\eta(\sigma-1)}{2(1-\eta)}\right]+\frac{\delta}{\rho}\cdot\lambda^c(\tilde{x}_i^{c*})^2. \quad \text{(III.9)}$$

将式（III.9）的左式和右式对 λ^c 求导，并将式（III.3）、式（III.5）代入可得：

$$\Rightarrow \frac{\tilde{x}_i^{f*}\cdot\dfrac{\partial \tilde{x}_i^{c*}}{\partial\lambda^c}-\tilde{x}_i^{c*}\cdot\dfrac{\partial \tilde{x}_i^{f*}}{\partial\lambda^c}}{(\tilde{x}_i^{f*})^2}=\frac{\delta}{\rho}\cdot\frac{\partial\lambda^c(\tilde{x}_i^{c*})^2}{\partial\lambda^c},$$

$$\begin{aligned}\Rightarrow \frac{\partial \tilde{x}_i^{f*}}{\partial\lambda^c}&=\frac{\tilde{x}_i^{f*}}{\tilde{x}_i^{c*}}\cdot\frac{\partial \tilde{x}_i^{c*}}{\partial\lambda^c}-\frac{\delta}{\rho}\cdot\frac{(\tilde{x}_i^{f*})^2}{\tilde{x}_i^{c*}}\cdot\frac{\partial\lambda^c(\tilde{x}_i^{c*})^2}{\partial\lambda^c}\\ &=\left\{\frac{\left(\dfrac{\rho}{2\delta}-\tilde{x}_i^{c*}\tilde{x}_i^{f*}\right)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}-1}{\dfrac{\rho}{2\delta}\cdot 2(1-\lambda^c)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2\lambda^c}\right\}\cdot\tilde{x}_i^{f*},\end{aligned} \quad \text{(III.10)}$$

$$\begin{aligned}\Rightarrow \frac{\partial(1-\lambda^c)\tilde{x}_i^{f*}}{\partial\lambda^c}&=(1-\lambda^c)\cdot\frac{\partial\tilde{x}_i^{f*}}{\partial\lambda^c}-\tilde{x}_i^{f*}\\ &=-\left\{\frac{(1-\lambda^c)\cdot\left[\dfrac{\rho}{2\delta}+\tilde{x}_i^{c*}\tilde{x}_i^{f*}\right]\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+1+\lambda^c}{\dfrac{\rho}{2\delta}\cdot 2(1-\lambda^c)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2\lambda^c}\right\}\cdot\tilde{x}_i^{f*}\leq 0,\end{aligned} \quad \text{(III.11)}$$

$$\begin{aligned}\Rightarrow \frac{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}{\partial\lambda^c}&=-(\tilde{x}_i^{f*})^2+2(1-\lambda^c)\tilde{x}_i^{f*}\cdot\frac{\partial\tilde{x}_i^{f*}}{\partial\lambda^c}\\ &=-\left\{\frac{2(1-\lambda^c)\tilde{x}_i^{c*}\tilde{x}_i^{f*}\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2}{\dfrac{\rho}{2\delta}\cdot 2(1-\lambda^c)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2\lambda^c}\right\}\cdot(\tilde{x}_i^{f*})^2\leq 0.\end{aligned} \quad \text{(III.12)}$$

也即，式（III.11）和式（III.12）可证得 $(1-\lambda^c)\tilde{x}_i^{f*}$ 和 $(1-\lambda^c)(\tilde{x}_i^{f*})^2$ 关于 λ^c 递减。

由式（III.9），代入式（III.5）中 $\lambda^c(\tilde{x}_i^{c*})^2$ 关于 λ^c 递增的关系，可得 $\tilde{x}_i^{c*}/\tilde{x}_i^{f*}$ 关于 λ^c 递增，反之，$\tilde{x}_i^{f*}/\tilde{x}_i^{c*}$ 关于 λ^c 递减。同时继续将式（III.1）代入可得：

$$\frac{\partial\left(\tilde{x}_i^{c*}/\tilde{x}_i^{f*}\right)}{\partial\lambda^c}=\frac{\partial\left(\tilde{x}_i^{c*}/\tilde{x}_i^{f*}\right)}{\partial\lambda^c(\tilde{x}_i^{c*})^2}\cdot\frac{\partial\lambda^c(\tilde{x}_i^{c*})^2}{\partial\lambda^c}=\frac{\delta}{\rho}\cdot\frac{\partial\lambda^c(\tilde{x}_i^{c*})^2}{\partial\lambda^c}\geq 0, \tag{III.13}$$

$$\left.\frac{\tilde{x}_i^{c*}}{\tilde{x}_i^{f*}}\right|_{\lambda^c=0}=\frac{\delta}{\tilde{\kappa}}\cdot\frac{1}{\sigma\rho}\cdot\left[1-\frac{\eta(\sigma-1)}{2(1-\eta)}\right],$$

$$\left.\frac{\tilde{x}_i^{c*}}{\tilde{x}_i^{f*}}\right|_{\lambda^c=1}=\frac{\delta}{\tilde{\kappa}}\cdot\frac{1}{\sigma\rho},$$

$$\Rightarrow\frac{\tilde{x}_i^{c*}}{\tilde{x}_i^{f*}}\in\left[\frac{\delta}{\tilde{\kappa}}\cdot\frac{1}{\sigma\rho}\cdot\left[1-\frac{\eta(\sigma-1)}{2(1-\eta)}\right],\quad\frac{\delta}{\tilde{\kappa}}\cdot\frac{1}{\sigma\rho}\right]. \tag{III.14}$$

III.1.2 分散经济的劳动力配置均衡结果

由分散经济的劳动力配置均衡结果：

将等式 $H_{yi}^*\cdot\left\{\left[\frac{(1-l_H)z_L}{l_Hz_H}\cdot\left(\frac{n_{\bar{L}}\chi}{H_{yi}^*}+1\right)\right]^{\frac{\zeta-1}{\zeta}}+1\right\}=n_{\bar{L}}\chi(\sigma-1)\cdot\left[\frac{\rho-n_{\bar{L}}}{n_{\bar{L}}+\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2}+1\right]$ 的两边对 λ^c 求导，并代入式（III.12）的 $\frac{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}{\partial\lambda^c}\leq 0$，可得：

$$\left\{\left[\frac{(1-l_H)z_L}{l_Hz_H}\cdot\left(\frac{n_{\bar{L}}\chi}{H_{yi}^*}+1\right)\right]^{-\frac{1}{\zeta}}\cdot\left[\frac{(1-l_H)z_L}{l_Hz_H}\cdot\left(\frac{1}{\zeta}\frac{n_{\bar{L}}\chi}{H_{yi}^*}+1\right)\right]+1\right\}\cdot\frac{\partial H_{yi}^*}{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}\cdot\frac{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}{\partial\lambda^c}$$
$$=-\frac{n_{\bar{L}}\chi(\sigma-1)\cdot(\rho-n_{\bar{L}})}{\left[n_{\bar{L}}+\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2\right]^2}\cdot\delta\cdot\frac{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}{\partial\lambda^c},$$

$$\Rightarrow\frac{\partial H_{yi}^*}{\partial\lambda^c}=\frac{\partial H_{yi}^*}{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}\cdot\frac{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}{\partial\lambda^c}=-\frac{\frac{n_{\bar{L}}\chi(\sigma-1)\cdot(\rho-n_{\bar{L}})}{\left[n_{\bar{L}}+\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2\right]^2}\cdot\delta\cdot\frac{\partial(1-\lambda^c)(\tilde{x}_i^{f*})^2}{\partial\lambda^c}}{\left[\frac{(1-l_H)z_L}{l_Hz_H}\cdot\left(\frac{n_{\bar{L}}\chi}{H_{yi}^*}+1\right)\right]^{-\frac{1}{\zeta}}\cdot\left[\frac{(1-l_H)z_L}{l_Hz_H}\cdot\left(\frac{1}{\zeta}\frac{n_{\bar{L}}\chi}{H_{yi}^*}+1\right)\right]+1}\geq 0. \tag{III.15}$$

继续推导 L_{yi}^*、$H_{et}^*:H_{yt}^*$ 和 N_t^* 的表达式，求导可得：

$$\frac{\partial L_{yi}^*}{\partial\lambda^c}=\frac{\partial L_{yi}^*}{\partial H_{yi}^*}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}=\frac{1-l_H}{l_H}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\geq 0, \tag{III.16}$$

$$\frac{\partial(H_{et}^*:H_{yt}^*)}{\partial\lambda^c}=\frac{\partial(H_{et}^*:H_{yt}^*)}{\partial H_{yi}^*}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}=-\frac{n_{\bar{L}}\chi}{(H_{yi}^*)^2}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\leq 0, \tag{III.17}$$

$$\frac{\partial N_t^*}{\partial\lambda^c}=\frac{\partial N_t^*}{\partial H_{yi}^*}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}=-\frac{l_H\bar{L}_0\cdot e^{n_{\bar{L}}\cdot t}}{\left(n_{\bar{L}}\chi+H_{yi}^*\right)^2}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\leq 0. \tag{III.18}$$

这意味着，企业数据共享将产生熟练劳动力再配置效应：随着 λ^c 增加，企业共享数据值 $(1-\lambda^c)\tilde{x}_i^{f*}$ 越少，潜在进入者开展创新活动后成功取代在位企业的概率 $\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2$ 越

低。故熟练劳动力选择投入新产品创新环节的比例越少（$H_{et}^{*}\big/H_{yt}^{*}$ 越低），产品种类的数量越少（N_t^* 越低），同时将使得单个企业的生产规模增大（H_{yi}^* 和 L_{yi}^* 均越高）。

III.1.3 分散经济的经济总量均衡结果

由分散经济的经济总产出均衡结果，并代入式（III.4）的 $\frac{\partial\lambda^c\tilde{x}_i^{c*}}{\partial\lambda^c}\geq 0$ 和式（III.11）的 $\frac{\partial(1-\lambda^c)\tilde{x}_i^{f*}}{\partial\lambda^c}\leq 0$，代入式（III.15）至式（III.18），求导可得：

$$\frac{\partial Y_t^*}{\partial\left[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]}=\frac{\eta}{1-\eta}\cdot\frac{Y_t^*}{\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}}>0,$$

$$\frac{\partial Y_t^*}{\partial[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}]}\cdot\frac{\partial\lambda^c\tilde{x}_i^{c*}}{\partial\lambda^c}\geq 0,$$

$$\frac{\partial Y_t^*}{\partial[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}]}\cdot\frac{\partial(1-\lambda^c)\tilde{x}_i^{f*}}{\partial\lambda^c}\leq 0,$$

$$\begin{aligned}&\left[\frac{\partial Y_t^*}{\partial N_0^*}\cdot\frac{\partial N_0^*}{\partial H_{yi}^*}+\frac{\partial Y_t^*}{\partial L_{yi}^*}\cdot\frac{\partial L_{yi}^*}{\partial H_{yi}^*}+\frac{\partial Y_t^*}{\partial H_{yi}^*}\right]\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\\&=\left\{-\left(\frac{1}{\sigma-1}+\frac{1}{1-\eta}\right)\cdot\frac{Y_t^*}{N_0^*}\cdot\frac{l_H\bar{L}_0}{\left(n_{\bar{L}}\chi+H_{yi}^*\right)^2}+\frac{Y_t^*}{(L_{yi}^*z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yi}^*z_H)^{\frac{\zeta-1}{\zeta}}}\cdot\begin{bmatrix}(L_{yi}^*z_L)^{\frac{\zeta-1}{\zeta}-1}\cdot\frac{(1-l_H)z_L}{l_H}\\+(H_{yi}^*z_H)^{\frac{\zeta-1}{\zeta}-1}\cdot z_H\end{bmatrix}\right\}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\\&=-\left[\frac{\eta}{1-\eta}+\frac{1}{\sigma-1}\cdot\frac{\rho-n_{\bar{L}}}{\rho+\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2}\right]\cdot\frac{Y_t^*}{n_{\bar{L}}\chi+H_{yi}^*}\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\leq 0.\end{aligned}$$

III.2 比较分散经济均衡和社会计划者均衡

为聚焦本文的研究结论，由于数值模拟中的相关参数计算可得两类数据共享的成本强度参数在 $\frac{\tilde{\kappa}}{\delta}\leq\frac{1}{\rho\sigma}\left[1-\frac{\eta(\sigma-1)}{1-\eta}\right]$ 的取值范围内，因此下文将基于 $\frac{\tilde{\kappa}}{\delta}\leq\frac{1}{\rho\sigma}\left[1-\frac{\eta(\sigma-1)}{1-\eta}\right]$ 的取值条件继续开展推导证明过程。此时式（III.6）、式（III.8）、式（III.14）可得：

$$\frac{\tilde{\kappa}}{\delta}\leq\frac{1}{\rho\sigma}\left[1-\frac{\eta(\sigma-1)}{1-\eta}\right]\quad\Rightarrow\quad\frac{\partial\tilde{x}_i^{c*}}{\partial\lambda^c}\leq 0,\quad\frac{\partial\tilde{x}_i^{*}}{\partial\lambda^c}\geq 0,\quad\frac{\tilde{x}_i^{c*}}{\tilde{x}_i^{f*}}\geq\frac{\eta(\sigma-1)}{2\left[(1-\eta)-\eta(\sigma-1)\right]}+1>1.\tag{III.19}$$

也即，消费者共享私有数据的意愿高于企业。进一步结合式（III.5）和式（III.10）可得：

$$\Rightarrow\frac{\partial\tilde{x}_i^{f*}}{\partial\lambda^c}=\frac{\tilde{x}_i^{f*}}{\tilde{x}_i^{c*}}\cdot\frac{\partial\tilde{x}_i^{c*}}{\partial\lambda^c}-\frac{\delta}{\rho}\cdot\frac{(\tilde{x}_i^{f*})^2}{\tilde{x}_i^{c*}}\cdot\frac{\partial\lambda^c(\tilde{x}_i^{c*})^2}{\partial\lambda^c}\leq 0.\tag{III.20}$$

也即，式（III.19）和式（III.20）可证得当满足 $\frac{\tilde{\kappa}}{\delta}\leq\frac{1}{\rho\sigma}\left[1-\frac{\eta(\sigma-1)}{1-\eta}\right]$ 条件时，$\tilde{x}_i^{c*}$ 和 $\tilde{x}_i^{f*}$ 关于 λ^c 递减，$\tilde{x}_i^{*}$ 关于 λ^c 递增。

III.2.1 比较数据共享均衡结果

由社会计划者的数据共享均衡结果 $2\tilde{\kappa}\cdot\tilde{x}_i^{c,sp}=\dfrac{\eta}{(1-\eta)\cdot\left[\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)\right]}$ 可得：

$$\Rightarrow\begin{cases}\tilde{x}_i^{c,sp}\big|_{\lambda^c=0}=\dfrac{\eta}{2\tilde{\kappa}\cdot(1-\eta)},\\ \tilde{x}_i^{c,sp}\big|_{\lambda^c=1}=\sqrt{\dfrac{\eta}{2\tilde{\kappa}\cdot(1-\eta)}},\end{cases}\tag{III.21}$$

$$\Rightarrow\frac{\partial\tilde{x}_i^{c,sp}}{\partial\lambda^c}=\frac{\tilde{x}_i^{c,sp}(1-\tilde{x}_i^{c,sp})}{2\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)}\geq0,\tag{III.22}$$

$$\Rightarrow\frac{\partial\lambda^c\tilde{x}_i^{c,sp}}{\partial\lambda^c}=\tilde{x}_i^{c,sp}+\lambda^c\cdot\frac{\partial\tilde{x}_i^{c,sp}}{\partial\lambda^c}=\frac{\tilde{x}_i^{c,sp}+\lambda^c\cdot(\tilde{x}_i^{c,sp})^2}{2\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)}\geq0,\tag{III.23}$$

$$\Rightarrow\frac{\partial\lambda^c(\tilde{x}_i^{c,sp})^2}{\partial\lambda^c}=(\tilde{x}_i^{c,sp})^2+2\lambda^c\tilde{x}_i^{c,sp}\cdot\frac{\partial\tilde{x}_i^{c,sp}}{\partial\lambda^c}=\frac{(1+\lambda^c)(\tilde{x}_i^{c,sp})^2}{2\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)}\geq0,\tag{III.24}$$

$$\Rightarrow\frac{\tilde{x}_i^{c,sp}}{\tilde{x}_i^{f,sp}}=\frac{\tilde{x}_i^{c,sp}}{1}\leq1,\tag{III.25}$$

$$\Rightarrow\frac{\partial\left(\tilde{x}_i^{c,sp}/\tilde{x}_i^{f,sp}\right)}{\partial\lambda^c}=\frac{\partial\left(\tilde{x}_i^{c,sp}/1\right)}{\partial\lambda^c}=\frac{\partial\tilde{x}_i^{c,sp}}{\partial\lambda^c}=\frac{\tilde{x}_i^{c,sp}(1-\tilde{x}_i^{c,sp})}{2\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)}\geq0.\tag{III.26}$$

其中，式（III.21）和式（III.22）的存在条件是 $\tilde{\kappa}\geq\dfrac{\eta}{2(1-\eta)}$。

比较消费者数据共享程度，结合式（III.5）和式（III.24）可得：

$$\begin{cases}\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]\Big|_{\lambda^c=0}=0,\\ \lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]\Big|_{\lambda^c=1}=\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]\Big|_{\lambda^c=1}=-\dfrac{\eta}{2\tilde{\kappa}\sigma\cdot(1-\eta)}<0,\end{cases}\tag{III.27}$$

$$\frac{\partial\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]}{\partial\lambda^c}=\frac{\dfrac{\rho}{2\delta}\cdot2\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}\cdot(\tilde{x}_i^{c*})^2}{\dfrac{\rho}{2\delta}\cdot2(1-\lambda^c)\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}+2\lambda^c}-\frac{(1+\lambda^c)(\tilde{x}_i^{c,sp})^2}{2\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)},\tag{III.28}$$

$$\begin{cases}\dfrac{\partial\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]}{\partial\lambda^c}\Bigg|_{\lambda^c=0}=\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]\Big|_{\lambda^c=0}=\dfrac{\delta}{\tilde{\kappa}^2\cdot\sigma^2\rho}\cdot\dfrac{\eta(\sigma-1)}{2(1-\eta)}\left[1-\dfrac{\eta(\sigma-1)}{2(1-\eta)}\right]-\left[\dfrac{\eta}{2\tilde{\kappa}\cdot(1-\eta)}\right]^2\\ \qquad=\begin{cases}>0,\quad if\ \ \delta>\dfrac{\sigma^2}{(\sigma-1)^2}\cdot\dfrac{\rho\eta(\sigma-1)}{2(1-\eta)-\eta(\sigma-1)},\\ \leq0,\quad if\ \ \delta\leq\dfrac{\sigma^2}{(\sigma-1)^2}\cdot\dfrac{\rho\eta(\sigma-1)}{2(1-\eta)-\eta(\sigma-1)},\end{cases}\\ \dfrac{\partial\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]}{\partial\lambda^c}\Bigg|_{\lambda^c=1}=\left\{\dfrac{\rho}{2\delta}\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+(\tilde{x}_i^{c*})^2\right\}^{-1}\cdot(\tilde{x}_i^{c*})^2-\tilde{x}_i^{c,sp}\right\}\Bigg|_{\lambda^c=1}\\ \qquad=\left\{\left\{\dfrac{\rho}{2\delta}\cdot\left\{\dfrac{1}{2\sigma\tilde{\kappa}}\left[1-\dfrac{\eta(\sigma-1)}{1-\eta}\right]+(\tilde{x}_i^{c*})^2\right\}^{-1}-1\right\}\cdot(\tilde{x}_i^{c*})^2-\left[\tilde{x}_i^{c,sp}-(\tilde{x}_i^{c*})^2\right]\right\}\Bigg|_{\lambda^c=1}<0.\end{cases}\tag{III.29}$$

其中，$\left.\frac{\partial\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]}{\partial\lambda^c}\right|_{\lambda^c=1}<0$ 的推导需代入 $\frac{\rho}{2\delta}\cdot\left\{\frac{1}{2\sigma\tilde{\kappa}}\left[1-\frac{\eta(\sigma-1)}{1-\eta}\right]+\lambda^c(\tilde{x}_i^{c*})^2\right\}^{-1}-1\le 0$，

以及 $-\left[\tilde{x}_i^{c,sp}-(\tilde{x}_i^{c*})^2\right]\Big|_{\lambda^c=1}=\frac{\eta(\sigma-1)}{\tilde{\kappa}\sigma\cdot 2(1-\eta)}-\sqrt{\frac{\eta}{2\tilde{\kappa}\cdot(1-\eta)}}\le\frac{\sigma-1}{\sigma}\frac{\eta}{2\tilde{\kappa}\cdot(1-\eta)}-\frac{\eta}{2\tilde{\kappa}\cdot(1-\eta)}<0$。

由式（III. 27）至式（III. 29）可得，当 $\delta>\frac{\sigma^2}{(\sigma-1)^2}\cdot\frac{\rho\eta(\sigma-1)}{2(1-\eta)-\eta(\sigma-1)}$ 时，$\lambda^c\cdot\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]$ 随着 λ^c 增加而先增大后减小。此时分散经济均衡存在消费者数据共享过多问题，且消费者隐私过度泄露程度随着 λ^c 增大而先恶化后缓解：存在两个临界比例 $0<\underline{\lambda}^c<\overline{\lambda}^c<1$，满足：

$$\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]=\begin{cases}\ge 0, & if\ \lambda^c\in[0,\overline{\lambda}^c],\\ <0, & if\ \lambda^c\in(\overline{\lambda}^c,1],\end{cases}\quad \frac{\partial\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]}{\partial\lambda^c}=\begin{cases}\ge 0, & if\ \lambda^c\in[0,\underline{\lambda}^c],\\ <0, & if\ \lambda^c\in(\underline{\lambda}^c,1].\end{cases}\tag{III. 30}$$

当 $\delta\le\frac{\sigma^2}{(\sigma-1)^2}\cdot\frac{\rho\eta(\sigma-1)}{2(1-\eta)-\eta(\sigma-1)}$ 时，$\lambda^c\cdot\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]$ 随着 λ^c 增大而减小。此时分散经济均衡存在消费者数据共享不足问题，且消费者隐私泄露程度随着 λ^c 增大而减弱，满足：

$$\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]\le 0,\qquad \frac{\partial\lambda^c\left[(\tilde{x}_i^{c*})^2-(\tilde{x}_i^{c,sp})^2\right]}{\partial\lambda^c}\le 0.\tag{III. 31}$$

比较整体数据的平均共享程度。由 $\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}=\frac{\eta(\sigma-1)}{2\tilde{\kappa}(1-\eta)\sigma\cdot\tilde{x}_i^{c*}}$ 和 $\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)\tilde{x}_i^{f,sp}=\frac{\eta}{2\tilde{\kappa}(1-\eta)\cdot\tilde{x}_i^{c,sp}}$，并代入 $\tilde{x}_i^{c*}\ge\tilde{x}_i^{f*}$ 和 $\tilde{x}_i^{c,sp}\le\tilde{x}_i^{f,sp}=1$ 可得：

$$\begin{cases}\left(\tilde{x}_i^{*}-\tilde{x}_i^{sp}\right)\Big|_{\lambda^c=0}=\left(\tilde{x}_i^{f*}-\tilde{x}_i^{f,sp}\right)\Big|_{\lambda^c=0}=\tilde{x}_i^{f*}\Big|_{\lambda^c=0}-1=\sqrt{\frac{\rho\eta(\sigma-1)}{\delta\cdot\left[2(1-\eta)-\eta(\sigma-1)\right]}}-1\le 0,\\ \left(\tilde{x}_i^{*}-\tilde{x}_i^{sp}\right)\Big|_{\lambda^c=1}=\left(\tilde{x}_i^{c*}-\tilde{x}_i^{c,sp}\right)\Big|_{\lambda^c=1}=\sqrt{\frac{\eta(\sigma-1)}{\tilde{\kappa}\sigma\cdot 2(1-\eta)}}-\sqrt{\frac{\eta}{2\tilde{\kappa}\cdot(1-\eta)}}<0,\end{cases}\tag{III. 32}$$

$$\begin{aligned}\frac{\partial\tilde{x}_i^{*}}{\partial\lambda^c}&=\frac{\partial\left[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]}{\partial\lambda^c}=-\frac{\eta(\sigma-1)}{2\tilde{\kappa}(1-\eta)\sigma\cdot(\tilde{x}_i^{c*})^2}\cdot\frac{\partial\tilde{x}_i^{c,*}}{\partial\lambda^c}\ge 0,\\ \frac{\partial\tilde{x}_i^{sp}}{\partial\lambda^c}&=\frac{\partial\left[\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)\tilde{x}_i^{f,sp}\right]}{\partial\lambda^c}=-\frac{\eta}{2\tilde{\kappa}(1-\eta)\cdot(\tilde{x}_i^{c,sp})^2}\cdot\frac{\partial\tilde{x}_i^{c,sp}}{\partial\lambda^c}\le 0,\\ &\Rightarrow\frac{\partial\left(\tilde{x}_i^{*}-\tilde{x}_i^{sp}\right)}{\partial\lambda^c}\ge 0,\qquad\Rightarrow\frac{\partial\left(\tilde{x}_i^{*}/\tilde{x}_i^{sp}\right)}{\partial\lambda^c}\ge 0.\end{aligned}\tag{III. 33}$$

也即，式（III. 32）和式（III. 33）表明分散经济均衡存在整体数据平均共享不足问题，该平均共享不足问题随着 λ^c 增大而减弱。

III. 2. 2 比较经济总量均衡结果

比较经济总产出可得：

$$\frac{Y_0^*}{Y_0^{sp}}=\frac{\left[\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]^{\frac{\eta}{1-\eta}}\cdot N_0^{*\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left[(L_{yi}^*z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yi}^*z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}}{\left[\lambda^c\tilde{x}_i^{c,sp}+(1-\lambda^c)\tilde{x}_i^{f,sp}\right]^{\frac{\eta}{1-\eta}}\cdot N_0^{sp\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left[(L_{yi}^{sp}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yi}^{sp}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}}\le 1,$$

$$\frac{\partial Y_0^*/Y_0^{sp}}{\partial\lambda^c}=\frac{\partial\left\{\left(\tilde{x}_i^*/\tilde{x}_i^{sp}\right)^{\frac{\eta}{1-\eta}}\cdot N_0^{*\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left[(L_{yi}^*z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yi}^*z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}\right\}}{\left(\partial\lambda^c\right)\cdot N_0^{sp\frac{1}{\sigma-1}+\frac{1}{1-\eta}}\cdot\left[(L_{yi}^{sp}z_L)^{\frac{\zeta-1}{\zeta}}+(H_{yi}^{sp}z_H)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}},\tag{III.34}$$

$$=\frac{Y_0^*}{Y_0^{sp}}\cdot\left\{\frac{\eta}{1-\eta}\cdot\frac{\partial\left(\tilde{x}_i^*/\tilde{x}_i^{sp}\right)/\partial\lambda^c}{\tilde{x}_i^*/\tilde{x}_i^{sp}}-\frac{1}{n_{\bar{L}}\chi+H_{yi}^*}\left[\frac{\eta}{1-\eta}+\frac{1}{\sigma-1}\cdot\frac{\rho-n_{\bar{L}}}{\rho+\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2}\right]\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\right\}.$$

其中，代入式（III.33）可得两类数据共享产生要素深化效应为 $\frac{\eta}{1-\eta}\cdot\frac{\partial\left(\tilde{x}_i^*/\tilde{x}_i^{sp}\right)/\partial\lambda^c}{\tilde{x}_i^*/\tilde{x}_i^{sp}}\ge 0$，代入式（III.15）可得企业数据共享的熟练劳动力再配置效应为 $-\frac{1}{n_{\bar{L}}\chi+H_{yi}^*}\left[\frac{\eta}{1-\eta}+\frac{1}{\sigma-1}\cdot\frac{\rho-n_{\bar{L}}}{\rho+\delta(1-\lambda^c)(\tilde{x}_i^{f*})^2}\right]\cdot\frac{\partial H_{yi}^*}{\partial\lambda^c}\le 0$。

比较社会福利可得：

$$U^*(c_t^*)=U^{sp}(c_t^{sp}\cdot\mathbf{\Omega}),$$

$$\Rightarrow\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0^*}{\bar{L}_0}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^{c*})^2+\frac{1}{\rho-n_{\bar{L}}}\cdot\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)\cdot n_{\bar{L}}\right]$$
$$=\frac{\bar{L}_0}{\rho-n_{\bar{L}}}\cdot\left[\ln\left(\frac{Y_0^{sp}}{\bar{L}_0}\cdot\mathbf{\Omega}\right)-\tilde{\kappa}\lambda^c(\tilde{x}_{it}^{c,sp})^2+\frac{1}{\rho-n_{\bar{L}}}\cdot\left(\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}\right)\cdot n_{\bar{L}}\right],$$

$$\Rightarrow\mathbf{\Omega}(*,sp)=\frac{Y_0^*}{Y_0^{sp}}\cdot\exp\left\{-\tilde{\kappa}\left[\lambda^c(\tilde{x}_{it}^{c*})^2-\lambda^c(\tilde{x}_{it}^{c,sp})^2\right]\right\}.$$

其中，$\frac{Y_0^*}{Y_0^{sp}}$ 关于 λ^c 的关系式参见式（III.34），而 $\lambda^c(\tilde{x}_i^{c*})^2-\lambda^c(\tilde{x}_i^{c,sp})^2$ 关于 λ^c 的关系式参见式（III.30）和式（III.31）。

附录 IV 参数的敏感性分析

数值模拟的基础是参数取值。一方面，广泛受认可且合理科学的参数取值是数值模拟的必要前提。另一方面，允许参数（尤其是没有中国实际经济数据的支撑时）在合理范围内波动且不影响主要结论是研究可信度和稳健性的必备要求。因此，附录 IV 将对各参数进行敏感性分析。图 IV1 至图 IV5 分别是对时间贴现率 ρ 、产品替代弹性 σ 、两类劳动力产出替代弹性 ζ 、数据要素重要性 η 、熟练劳动力生产效率 z_H 的参数取值的敏感性分析。数值模拟结果表明，当这些参数在合理范围内取值变化时，本文核心结论“若以社会计划者最优均衡作为比较基准，则分散经济均衡的社会福利效率与消费者数据占比呈现非线性关系”依然成立。

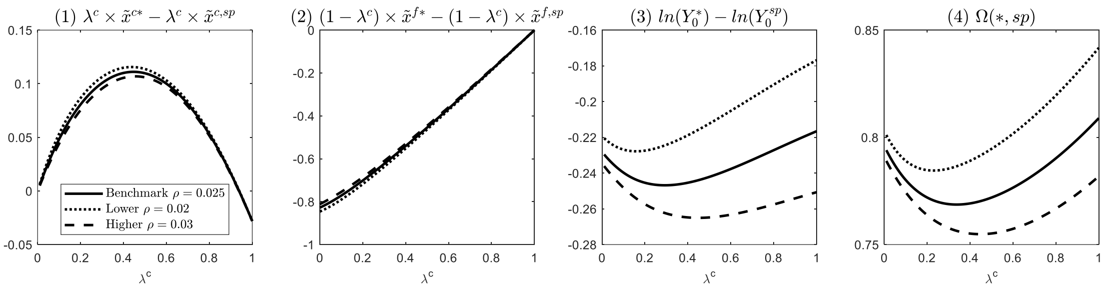

图 IV1 对时间贴现率 ρ 的敏感性分析

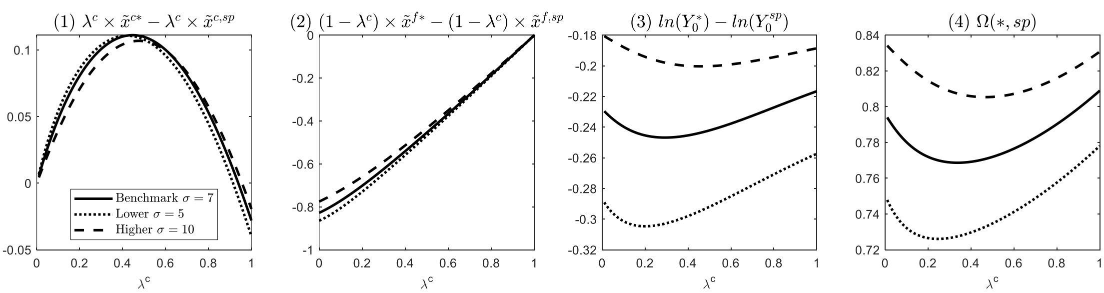

图 IV2 对产品替代弹性 σ 的敏感性分析

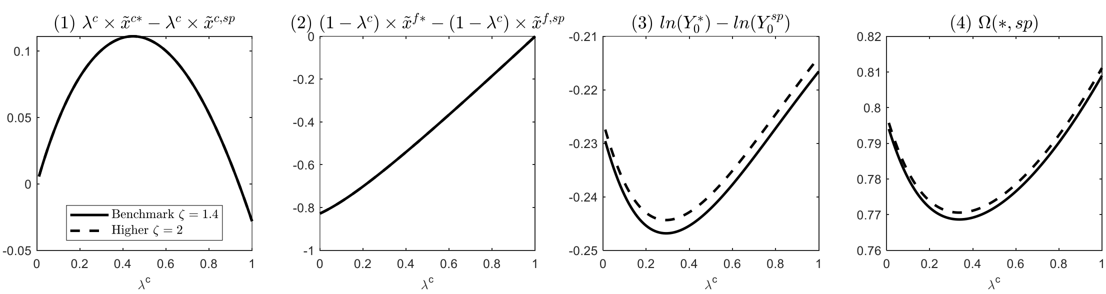

图 IV3 对两类劳动力产出替代弹性 ζ 的敏感性分析

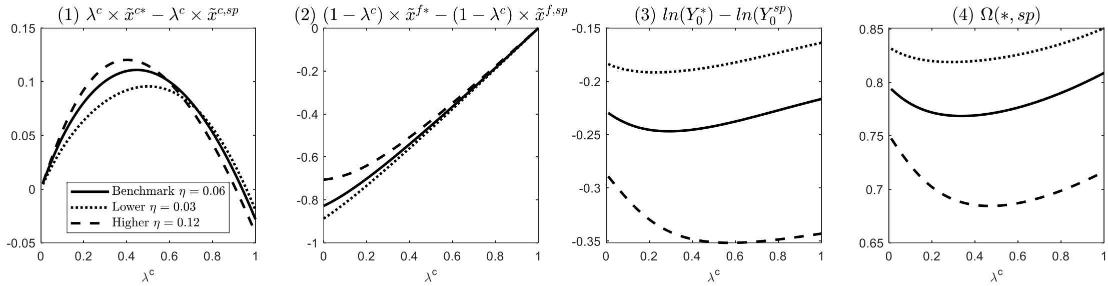

图 IV4 对数据要素重要性 η 的敏感性分析

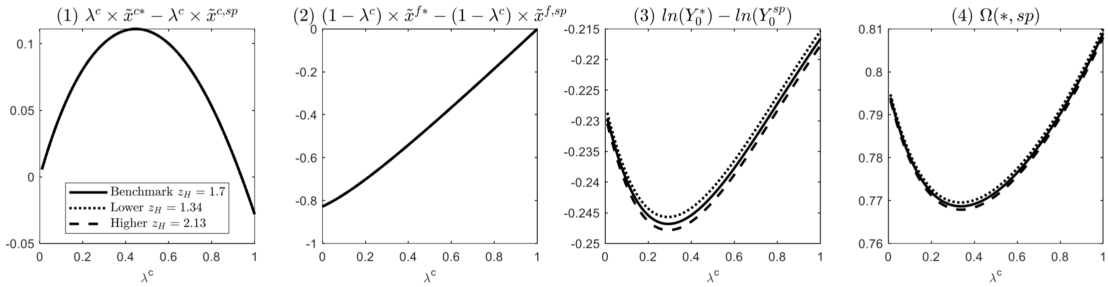

图 IV5 对熟练劳动力生产效率 z_H 的敏感性分析

附录V 拓展理论模型及数值模拟结果

V.1 拓展理论模型:修改假设

正文理论模型中假设在位企业被新企业取代的概率只与产品生产过程的企业数据有关，而与产品消费过程的消费者数据无关。也即原假设是，由于社会数据池的企业信息泄露问题，令每期在位企业被新企业取代的概率服从泊松到达率 $\delta(1-\lambda^{c})\cdot(\tilde{x}_{it}^{f})^{2}$。但是，借鉴 Jones and Tonetti（2020）的思路，一种可拓展的合理假设是，在位企业被新企业取代的概率同时与企业数据泄露和消费者数据泄露有关。

据此，附录 V 将对原假设进行修改，令新假设是：每期在位企业被新企业取代的概率服从泊松到达率 $\delta\cdot[\gamma\cdot\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\tilde{x}_{it}^{f}]^{2}$，也即被取代的概率同时受到消费者数据泄露和企业数据泄露的影响。此时，创造性破坏的强度为参数 δ，数据泄露的规模系数为 1，数据共享使用比例的指数是边际递增的二次函数形式。同时，新增引入参数 γ 满足取值范围 $0\le\gamma\le1$，表示消费者数据泄露对企业被创造性破坏成本的作用力度，应当不高于企业数据泄露的作用力度。拓展理论模型的其他假设与正文的理论模型保持一致。

因此，第 i 种产品的代表性企业价值 V_{it} 的最大化问题可以改写为：

$$r_{t}V_{it}=\max_{\substack{\{L_{yit},H_{yit},\tilde{D}_{t}^{c},\tilde{D}_{t}^{f},\\ D_{it}^{c},x_{it}^{f},\tilde{x}_{it}^{f}\}}}\left\{\begin{array}{l}\left[p_{Dit}^{c}\cdot\lambda^{c}x_{it}^{c}+\tilde{p}_{Dit}^{c}\cdot\lambda^{c}\tilde{x}_{it}^{c}+\left(Y_{t}/Y_{it}\right)^{1/\sigma}\right]\cdot Y_{it}-w_{Lt}L_{yit}-w_{Ht}H_{yit}-\tilde{p}_{Dt}^{mc}\cdot\tilde{D}_{t}^{c}\\ -\tilde{p}_{Dt}^{mf}\cdot\tilde{D}_{t}^{f}-p_{Dit}^{c}\cdot D_{it}^{c}+\tilde{p}_{Dit}^{f}\cdot\tilde{x}_{it}^{f}\cdot(1-\lambda^{c})R_{it}+\dot{V}_{it}-\delta\cdot\left[\gamma\cdot\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\tilde{x}_{it}^{f}\right]^{2}\cdot V_{it}\end{array}\right\}.$$

产品市场的自由进出条件可以改写为：

$$\chi w_{Ht}=V_{it}+\frac{\int_{0}^{N_{t}}\delta\cdot\left[\gamma\cdot\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\tilde{x}_{it}^{f}\right]^{2}\cdot V_{it}di}{\dot{N}_{t}}=V_{it}+\frac{\delta\cdot\left[\gamma\cdot\lambda^{c}\tilde{x}_{it}^{c}+(1-\lambda^{c})\tilde{x}_{it}^{f}\right]^{2}\cdot V_{it}}{g_{N}}.$$

V.2 拓展理论模型:经济均衡结果

对社会计划者而言，私有数据共享引发的企业被创造性破坏和商业窃取，实际上是在位企业价值向新进入企业价值的“无损”转移，故无论如何设定在位企业被新企业取代概率的相关假设，均不影响社会计划者的均衡配置结果，仅会通过影响代表性企业最优化问题和产品市场自由进出条件而影响分散经济均衡结果。

对分散经济而言，结合代表性家庭、代表性企业、数据中介的最优化条件和市场自由进出条件，可得当经济体的市场规模足够大（$N_{t}\to\infty$）时，分散经济均衡将无限收敛于如下稳态结果（其他均衡结果的表达式与正文的式（6）至式（11）相同）：

$$2\tilde{\kappa}\cdot\tilde{x}_{i}^{c*}=\frac{2\delta\cdot\left[\gamma\cdot\lambda^{c}\tilde{x}_{i}^{c*}+(1-\lambda^{c})\tilde{x}_{i}^{f*}\right]}{\sigma\delta\left[\gamma\cdot\lambda^{c}\tilde{x}_{i}^{c*}+(1-\lambda^{c})\tilde{x}_{i}^{f*}\right]^{2}+\sigma\rho}=\frac{\eta(\sigma-1)}{(1-\eta)\sigma\cdot\left[\lambda^{c}\tilde{x}_{i}^{c*}+(1-\lambda^{c})\tilde{x}_{i}^{f*}\right]},\qquad(\text{V}.1)$$

$$H_{yi}^{*}\cdot\left\{\left[\frac{(1-l_{H})z_{L}}{l_{H}z_{H}}\cdot\left(\frac{n_{\bar{L}}\chi}{H_{yi}^{*}}+1\right)\right]^{\frac{\zeta-1}{\zeta}}+1\right\}=n_{\bar{L}}\chi(\sigma-1)\cdot\left\{\frac{\rho-n_{\bar{L}}}{n_{\bar{L}}+\delta\cdot\left[\gamma\cdot\lambda^{c}\tilde{x}_{i}^{c*}+(1-\lambda^{c})\tilde{x}_{i}^{f*}\right]^{2}}+1\right\}.\qquad(\text{V}.2)$$

其中，式（V.1）刻画了数据共享的边际成本等于边际收益。与正文原假设下的式（4）的不同之处在于，消费者数据共享和企业数据共享共同导致了在位企业被取代的概率边际提高 $2\delta\cdot\left[\gamma\cdot\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]$，同时平均存活寿命降低也导致在位企业的市场价值降低（V_i 正比于 $\dfrac{1}{\sigma\delta\left[\gamma\cdot\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]^2+\sigma\rho}$），故边际成本是 $\dfrac{2\delta\cdot\left[\gamma\cdot\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]}{\sigma\delta\left[\gamma\cdot\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}\right]^2+\sigma\rho}$。

由式（V.1）继续推导可得：

$$\Rightarrow \tilde{x}_i^{c*}=\frac{-(1-\lambda^c)\tilde{x}_i^{f*}+\sqrt{(1-\lambda^c)^2(\tilde{x}_i^{f*})^2+\dfrac{2\eta(\sigma-1)\lambda^c}{(1-\eta)\sigma\tilde{\kappa}}}}{2\lambda^c}, \tag{V.3}$$

$$\Rightarrow \tilde{x}_i^{f*}=\frac{\eta(\sigma-1)}{(1-\eta)\cdot(1-\lambda^c)2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-\frac{\lambda^c\tilde{x}_i^{c*}}{1-\lambda^c},$$

$$\Rightarrow \frac{\eta(\sigma-1)}{(1-\eta)\cdot 2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-(1-\gamma)\cdot\lambda^c\tilde{x}_i^{c*}=\frac{1}{2\sigma\tilde{\kappa}\tilde{x}_i^{c*}}-\sqrt{\frac{1}{\left(2\sigma\tilde{\kappa}\tilde{x}_i^{c*}\right)^2}-\frac{\rho}{\delta}}. \tag{V.4}$$

因此，$\tilde{x}_i^{c*}$ 和 $\tilde{x}_i^{f*}$ 将由式（V.3）与式（V.4）的等式结果计算得到。

V.3 拓展理论模型:数值模拟结果

基于在新假设下的分散经济均衡结果和社会计划者均衡结果、正文参数的基准取值以及新增参数 γ 在取值范围[0,1]内的三种不同典型取值：$\gamma=0$（左侧极端取值）、$\gamma=0.5$（折中取值）、$\gamma=1$（右侧极端取值），图 V1 绘制了分散经济均衡的社会福利效率 $\mathbf{\Omega}(*,sp)$。

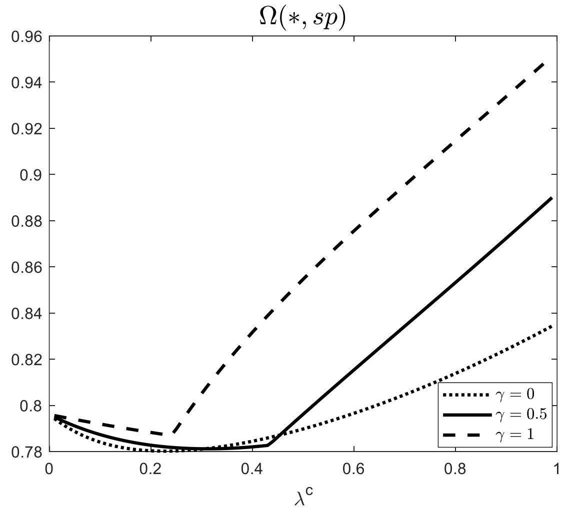

图 V1 拓展理论模型：分散经济均衡效率

分析图像可得，在新假设下的数值模拟结果依然支持本文结论：若以社会计划者最优均衡作为比较基准，则分散经济均衡的社会福利效率与消费者数据占比呈现非线性关系。同时，随着参数 γ 从 $\gamma=0$ 、到 $\gamma=0.5$ 、再到 $\gamma=1$,该非线性关系的右侧递增部分被加强得越显著。其经济学直觉是，随着消费者数据占比 λ^c 增大，消费者数据共享值 $\lambda^c\tilde{x}_i^{c*}$ 增多，企业数据共享值 $(1-\lambda^c)\tilde{x}_i^{f*}$ 减少。在新假设和原假设下的熟练劳动力再配置效应有所不同：在

原假设下，仅考虑企业共享数据值减少，在位企业被新企业取代的概率 $\delta(1-\lambda^c)\cdot(\tilde{x}_i^{f*})^2$ 越低，产生抑制新产品创新的熟练劳动力再配置效应，进而降低经济总产出和社会福利水平。而在新假设下，新增考虑消费者数据共享值增多，在位企业被新企业取代的概率 $\delta\cdot[\gamma\cdot\lambda^c\tilde{x}_i^{c*}+(1-\lambda^c)\tilde{x}_i^{f*}]^2$ 越高，产生激发新产品创新的熟练劳动力再配置效应，进而提高经济总产出和社会福利水平。并且，这一新增的作用机制随着参数 γ 越大（进一步提高在位企业被新企业取代的概率）而被加强得越显著。

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