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Production experiences and market structure in R&D competition

Shun-Chiao Chang[a,*], Ho-Mou Wu[b]

[a]*Department of Economics, National Chi-Nan University, Taiwan*
[b]*School of Management, National Taiwan University, Taiwan*

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Abstract

In the R&D race the incumbent enjoys an advantage of learning from production experiences, but this important feature has not been incorporated into existing studies. Assuming that the technological knowledge is accumulated not only by R&D expenditures but also by production experiences, we study the properties of optimal investment strategies in a model with an incumbent and many identical challengers. After proving the existence of a unique Nash equilibrium in the R&D race, we demonstrate analytically that the likelihood of persistent leadership increases with production experiences of the incumbent but decreases with the number of challengers. Numerical analyses also establish that (i) the challengers always invest more than the incumbent and the difference increases with production experiences, the flow of monopoly profits and the number of challengers; and (ii) the likelihood of persistent leadership increases with the value of being the winner and the value of being a loser but decreases with expected waiting time of R&D innovation and the flow of monopoly profits. However, destructive innovations may still occur even when production experiences are allowed to play an important role in the R&D competition.

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*Corresponding author.
E-mail addresses: shunchiao@ncnu.edu.tw (S.-C. Chang), homouwu@mba.ntu.edu.tw (H.-M. Wu).

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1. Introduction

In this paper we develop a framework to understand the impact of production experiences on the R&D race and study whether the incumbent can prevail in the face of possibly destructive innovations. Although the effects of production experiences on production costs[1] and the structure of an industry[2] are in the literature, their impact on the R&D race has not been fully analyzed.

Production experiences can play to the advantages of the incumbent in the R&D competition. For example, Intel's experience of producing 486 CPU provided the company with an opportunity to supersede its major rivals in the development of the products of the next four generations – namely, Pentium I, Pentium II, Pentium III, and Pentium IV CPUs (Yu, 1999; Chang and Park, 2004). Similarly, Gruber's (1994) empirical studies also show that production experiences helped the incumbent to prevail in the patent race for the next-generation EPROMs products. Production experiences, however, may not be sufficient for the incumbent to sustain its edge over its competitors as more challengers emerge. In the optical passive components (OPC) industry for example, an increase of new competitors from Taiwan and Korea caused most of the Japanese and American incumbents to switch to the optical active components (OAC) industry. It shows that destructive innovations may occur as the R&D race becomes more competitive.

In a seminal paper, Reinganum (1982) studies the strategies of a number of identical firms engaged in R&D race. An increase in the number of challengers is shown to lead to an increase in each firm's R&D investment.[3] Reinganum (1983) also points out that the incumbent's R&D investment is less than that of the challenger when there is only one challenger and one incumbent and when the innovation process is uncertain. Furthermore, when there is one incumbent and a number of identical challengers and when the new-generation products are introduced to replace the obsolete products as in Reinganum (1985), the incumbent always invests less than the challengers since the incumbent lacks an incentive for R&D investments. Hence, the Schumpeterian 'process of destructive innovations' can occur in a sequence of innovations as the incumbent is overthrown by a more innovative challenger.[4]

In this paper, we include not only the impact of market competition but also the effects of accumulated production experiences on the R&D competition for the next generation product. We assume that each firm's hazard function governing the

[1] See Dick (1994), Benkard (2000), Park (2002) and Cabral and Leiblein (2001).

[2] See Gilbert and Newbery (1982), Dasgupta and Stiglitz (1988) and Reinganum (1983, 1985).

[3] Loury (1979) and Lee and Wilde (1980) also discuss the variations of each participant's investment intensities as the number of competitors changes. In addition, Choi (1991) and Malueg and Tsutsui (1997) analyze a patent race for several identical firms with an uncertain hazard rate governing the innovation process.

[4] The persistence of monopoly in the R&D race for uncertain process innovations is possible when the monopolist incumbent has a first-mover advantage to commit to an entry-deterring level of investment (see Gilbert and Newbery, 1982, 1984; Reinganum, 1984).

innovation process depends on its accumulated production experiences and the cumulative flow of R&D expenditures. We further assume that the incumbent and the challengers are heterogeneous. Only the incumbent can enjoy monopoly profit and accumulated knowledge from production experiences, while all participants have to pay a fixed cost at the outset as well as recurrent flow costs when the R&D competition is underway. Besides the probability distribution governing the timing of innovation, the incumbent's production experiences, the monopoly profit from the post-innovation market, the patent reward and the payoff to the loser will all affect each firm's equilibrium strategy in our model.

After proving the existence of a unique Nash equilibrium in this differential game (Theorem 1), we demonstrate analytically that the likelihood of persistent leadership increases with production experiences of the incumbent but decreases with the number of challengers (Theorem 2). It confirms the observation that production experiences can play to the advantage of the incumbent such as Intel. It also shows that destructive innovation is more likely to happen with increased market competition, providing an explanation for the OPC industry example mentioned before. However, it is difficult to obtain a closed-form solution of the optimal investment strategies from a set of nonlinear first-order partial differential equations. Numerical simulations are therefore conducted to explore the properties of equilibrium investment strategies. In the absence of production experience (called the benchmark case), we first confirm (in the appendix) that our numerical analyses reach the same conclusions as in Reinganum's (1983) model, when there is only one incumbent and one challenger.

Our numerical analyses further indicate that the incumbent invests less than the challenger when we include the influence of production experiences. This difference increases with production experiences since the challengers' investment rates increase with production experiences while the incumbent's investment rate decreases with production experiences (Result 1). The difference between the challengers' and the incumbent's investment rates decreases with the value of being a winner and a loser but increases with the expected waiting time of an innovation and the flow of monopoly profits (Result 2). With a large number of challengers, the persistent leadership in the R&D race is still possible if production experience of the incumbent is sufficiently large. Moreover, the likelihood of persistent leadership increases with the present value of being a winner and a loser but decreases with the expected waiting time of R&D innovation and the flow of monopoly profits (Result 3). The difference between the challengers' and the incumbent's investment rates increases with the number of challengers in the R&D race (Result 4).

The paper is organized as follows. The next section contains the model and the existence and uniqueness result (Theorem 1). In Section 3, we present the analytical result of Theorem 2 and other numerical analyses (Results 1–4). The proof of Theorem 1, discussion of our numerical method and confirmation of the benchmark case (in the absence of production experiences) are included in the appendix. Section 4 concludes the paper.

2. The model

2.1. Notations and framework

We assume that one incumbent (firm I) and n identical challengers ($c = 1, 2, 3, \ldots, n$) engage in a multistage patent race. The incumbent enjoys monopoly profit and production experiences from the previous generation product throughout each R&D competition. We assume that the incumbent produces one unit of product per unit of time and its technological knowledge accumulated from production experiences at time τ is proportional to its cumulative output. Hence, accumulated R&D knowledge is equal to $\theta\tau$, where θ $(\theta > 0)$ measures the marginal production experience.[5] In our model, production experiences affect each firm's winning and losing probability.

Following Reinganum (1982, 1985), we adopt a continuous-time framework. In each stage, $n+1$ participants compete by investing $u^j(\tau)$, $j = I, 1, 2, \ldots, n$ at time τ. Let $X^j(\tau)$ denote firm j's R&D capital with $\dot{X}^j(\tau) = u^j(\tau)$, where the dot means the time derivative. Let $X^I(\tau) + \theta\tau$ and $X^c(\tau)$ denote the incumbent's (firm I) and each challenger's (firm c) accumulated R&D knowledge at time τ, respectively. And let $X(\tau) = X^I(\tau) + \theta\tau + \sum_{c=1}^{n} X^c(\tau)$ denote the overall R&D knowledge at time τ. The value of R&D knowledge is common knowledge in the game. We further assume that the probability of firm j succeeding in innovation at or by time $\tau^j(X^I + \theta\tau, X^1, X^2, \ldots, X^n)$ follows an exponential distribution with parameter $\lambda > 0$, $\Pr\{\exists j \in \{I, 1, 2, \ldots, n\};\ \tau^j \leqslant \tau\} = 1 - \exp\{-\lambda X(\tau)\}$.[6] Thus, the instantaneous probability that the incumbent succeeds at time τ and its competitors lose by time τ is

$$\begin{aligned} &\Pr\{\tau^c > \tau, \text{ for all challengers } c = 1, 2, \ldots, n,\ \tau^I < \tau + \mathrm{d}\tau\} \\ &\quad = \exp\{-\lambda X(\tau)\}\lambda(u^I(\tau) + \theta)\,\mathrm{d}\tau. \end{aligned} \tag{1a}$$

Similarly, the instantaneous probability of the incumbent of being a loser at time τ is

$$\Pr\{\tau^I > \tau, \tau^c < \tau + \mathrm{d}\tau, \text{ for one firm } c\} = \exp\{-\lambda X(\tau)\}\lambda \sum_{c=1}^{n} u^c(\tau)\,\mathrm{d}\tau. \tag{1b}$$

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[5] Production experiences are considered as an important factor in the dynamic models of learning by doing (e.g. Stokey, 1988; Young, 1991, 1993; Auerswald et al., 2000). Although the impact may be nonlinear or dependent on some discount factor, we assume the current form in order to obtain sharper characterization. A more general form with the more recent experience having a larger impact will be left for further study.

[6] As $\Pr\{\tau^j \leqslant \tau\} = 1 - \exp\{-\lambda X(\tau)\} = S(\tau)$ is the survival function in duration analysis, the hazard function $h(\tau)$ has to satisfy $h(\tau) = -S'(\tau)/S(\tau)$, where $S'(\tau) = \mathrm{d}S(\tau)/\mathrm{d}\tau$. Therefore, $h(\tau)$ is a constant ($h(\tau) = \lambda$) under the exponential distribution with parameter $\lambda > 0$, as in Reignanum (1982, 1985). This requires the probability distribution of innovation success to be memoryless. A more general form can be considered in a further study.

The instantaneous probability that one of n challengers succeeds at time τ and the others fail by time τ is

$$\Pr\{\tau^j > \tau, \text{ for all firm } j,\ j \neq c,\ j = I, 1, \ldots, n \text{ and } \tau^c < \tau + d\tau\}$$
$$= \exp\{-\lambda X(\tau)\}\lambda u^c(\tau)\, d\tau. \tag{2a}$$

The instantaneous probability of one of n challengers of being a loser at time τ is

$$\Pr\{\tau^j < \tau + d\tau, \text{ for one firm } j,\ j \neq c,\ j = I,\ 1, \ldots, n \text{ and } \tau^c > \tau\}$$
$$= \exp\{-\lambda X(\tau)\}\left[\lambda\left(u^I(\tau) + \theta + \sum_{j=1, j\neq c}^{n} u^j(\tau)\right)\right] d\tau. \tag{2b}$$

In our dynamic model, we assume that all firms choose their investment strategies simultaneously to maximize expected payoff. The incumbent receives a flow of monopoly profit from the previous product at the constant rate of R while it has to pay a lump sum fixed cost, F, at the outset as well as a current flow cost, $(u^j)^2/2$, of R&D investment.[7] In our multistage dynamic game, each stage of R&D competition will end if any firm succeeds in innovation, and then the winner and the losers will receive the terminal value, υ^ω and υ^ℓ ($\upsilon^\omega > \upsilon^\ell$), respectively.[8] We further assume that the R&D game ends at time T when one of the firms succeeds in innovation. The date T can also be regarded as the doomsday at which the firms abandon the project entirely if they have not yet succeeded (see Reinganum, 1982). Hence the incumbent's expected payoff function for any strategy tuple $(u^I, u^1, u^2, \ldots, u^n)$ can be written as follows:

$$V^I(u^I, u^1, u^2, \ldots, u^n) = \int_0^T \left\{ e^{-\lambda X(\tau)}\left[\lambda\upsilon^\omega(u^I(\tau) + \theta) + \lambda\upsilon^\ell \sum_{c=1}^{n} u^c(\tau)\right] + e^{-r\tau}(R - (u^I(\tau))^2/2)\right\} d\tau - F. \tag{3}$$

Similarly, the challenger's expected payoff function is

$$V^c(u^I, u^1, u^2, \ldots, u^n) = \int_0^T \left\{ e^{-\lambda X(\tau)}\left[\lambda\upsilon^\omega u^c(\tau) + \lambda\upsilon^\ell\left(u^I(\tau) + \theta + \sum_{j=1, j\neq c}^{n} u^j(\tau)\right)\right] - e^{-r\tau}(u^c(\tau)^2/2)\right\} d\tau - F, \tag{4}$$

[7] The current flow cost of R&D investment can be generalized from the quadratic form to the cases with constant elasticity. Since the cost structure is not the main focus of our research, we adopt the quadratic form in this paper as in Reinganum (1982, 1985) and Malueg and Tsutsui (1997).

[8] As discussed in Reinganum (1985), the terminal values represent the values of continuing optimally in the patent race and may be treated as parameters in each stage when the sequence of innovation is finite due to technological reasons.

where T is the doomsday at which the earnings must be actualized for all participants. The first term of (3), $\exp\{-\lambda X(\tau)\}\lambda\upsilon^{\omega}(u^{I}(\tau)+\theta)\,\mathrm{d}\tau$, represents the expected winning reward of the incumbent; the second term of (3), $\exp\{-\lambda X(\tau)\}\lambda\upsilon^{\ell}\sum_{c=1}^{n}u^{c}(\tau)\,\mathrm{d}\tau$, is the incumbent's expected payoff when it becomes a loser; and $\mathrm{e}^{-\gamma\tau}R$ and $\mathrm{e}^{-\gamma\tau}(u^{I})^{2}/2$ of (3) represent the incumbent's discounted monopoly profits from previous-generation product and discounted investment costs, respectively. Similarly, we can find the meaning of different terms of Eq. (4). These payoff functions V^{j}, $j=I,1,2,\ldots,n$ will be used to define the equilibrium concept.

2.2. Nash equilibrium strategies

Following Reinganum (1982), we will consider only pure (closed-loop) strategies, assuming that each firm's instantaneous investment rate is bounded. For proving the existence of a unique Nash equilibrium in our differential game, we need the following assumption on the strategy space. For notational simplicity, let $X=(X^{I},X^{1},X^{2},\ldots,X^{n})$.

Assumption 1. The strategy space for firm j, $j=I,1,2,\ldots,n$, is $\Omega^{j}=\{u^{j}(\tau,X)\in[0,K]$ for some $K<\infty\}$, where for all $(\tau,X)\in[0,T]\times \mathrm{X}_{I}\times \mathrm{X}_{1}\times\cdots\times \mathrm{X}_{n}$, $u^{j}(\tau,X)$ is continuous in (τ,X) and satisfies the Lipschitz condition:

$$|u^{j}(\tau,X)-u^{j}(\tau,\bar{X})|\leqslant k(\tau)|X-\bar{X}|, \tag{5}$$

where $k(\tau)$ is a Lipschitz constant such that $k(\tau)=\sup_{(\tau,X^{I},X^{1},X^{2},\ldots,X^{n})}|\partial u^{j}/\partial X^{j}|$.

This assumption requires that the investment strategy $u^{j}(\tau,X)$ of each firm, belonging to a closed and bounded space, is continuous and differentiable. This is commonly assumed in the literature in differentiable game (see Friedman, 1971).

Definition 1. The $n+1$ firms' strategy tuple $(u_{*}^{I},u_{*}^{1},u_{*}^{2},\ldots,u_{*}^{n})$ is a Nash equilibrium if $u_{*}^{j}\in\Omega^{j}$ and $V^{j}(u_{*}^{I},u_{*}^{1},\ldots,u_{*}^{j-1},u_{*}^{j},u_{*}^{j+1},\ldots,u_{*}^{n})\geqslant V^{j}(u_{*}^{I},u_{*}^{1},\ldots,u_{*}^{j-1},u^{j},u_{*}^{j+1},\ldots,u_{*}^{n})$ for all $u^{j}\in\Omega^{j}$ and $j=I,1,2,\ldots,n$.

Integrating the first term of the payoff functions in (3) and (4) by parts, we can define the value functions at any time $s\in[0,T]$ as follows:

$$\begin{aligned}\Phi^{I}(s,X)=\int_{s}^{T}\Bigg\{&[\upsilon^{\omega}(1-\Delta_{I}^{\omega}(\tau))+\upsilon^{\ell}\Delta_{I}^{\omega}(\tau)]\Delta_{c}(\tau)\lambda\sum_{c=1}^{n}u_{*}^{c}(\tau)+\mathrm{e}^{-r\tau}\Delta(\tau)\\&\times\left(R-\tfrac{1}{2}(u^{I}(\tau))^{2}\right)\Bigg\}\mathrm{d}\tau+\upsilon^{\omega}(1-\Delta_{I}^{\omega}(T))\Delta_{c}(T)-F,\quad s\in[0,T],\end{aligned} \tag{6}$$

where $\Delta_I^{\omega}(\tau) = \exp\{-\lambda[X^I(\tau) + \theta\tau]\}, \Delta_c(\tau) = \exp\{-\lambda\sum_{c=1}^{n} X^c(\tau)\}$ and $\Delta(\tau) = \exp\{-\lambda[X^I(\tau) + \theta\tau + \sum_{c=1}^{n} X^c(\tau)]\}$.

$$\Phi^c(s, X) = \int_s^T \left\{ [\upsilon^{\omega}(1 - \Delta_c^{\omega}(\tau)) + \upsilon^{\ell}\Delta_c^{\omega}(\tau)]\Delta_I(\tau)\lambda\left(u_*^I(\tau) + \theta + \sum_{j\neq c,j=1}^{n} u^j\right) - \mathrm{e}^{-r\tau}\Delta(\tau)(\tfrac{1}{2}(u^c(\tau))^2) \right\} \mathrm{d}\tau + \upsilon^{\omega}(1 - \Delta_c^{\omega}(T))\Delta_I(T) - F, \tag{7}$$

where $\Delta_I(\tau) = \exp\{-\lambda[X^I(\tau) + \theta\tau + \sum_{j\neq c,j=1}^{n} X^j]\}$ and $\Delta_c^{\omega}(\tau) = \exp\{-\lambda X^c(\tau)\}$.

As shown in Friedman (1971, Theorem 4.3.2 (p. 141) and 8.2.2 (p. 292)), the Hamilton–Jacobi equations in our $n+1$-firm differential game reveal that all firms' optimal investment strategies must satisfy a system of Bellman equations (Eqs. (8) and (9)) subject to the respective terminal conditions (Eqs. (10) and (11)) given that Assumption 1 holds, and that (6) and (7) are continuously differentiable. This fact is used in the proof of Theorem 1 in Appendix A.

$$\Phi_\tau^I(\tau, X) + \max_{u^I\in[0,K]} \left\{ \Phi_{X^I}^I(\tau, X)u^I(\tau, X) + \sum_{c=1}^{n} \Phi_{X^C}^c(\tau, X)u_*^c(\tau, X) + [\upsilon^{\varpi}(1 - \Delta_I^{\omega}(\tau)) + \upsilon^{\ell}\Delta_I^{\omega}(\tau)]\Delta_c(\tau)\lambda\sum_{c=1}^{n} u_*^c(\tau) + \mathrm{e}^{-r\tau}\Delta(\tau)(R - \tfrac{1}{2}(u^I)^2) \right\} = 0, \tag{8}$$

$$\Phi_\tau^c(\tau, X) + \max_{u^c\in[0,K]} \left\{ \Phi_{X^c}^c(\tau, X)u^c((\tau, X)) + \Phi_{X^I}^I(\tau, X)u_*^I(\tau, X) + \sum_{j\neq c,j=1}^{n} \Phi_{X^j}^i(\tau, X)u_*^j(\tau, X) + [\upsilon^{\omega}(1 - \Delta_c^{\omega}(\tau)) + \upsilon^{\ell}\Delta_c^{\omega}(\tau)] \times \Delta_I(\tau)\lambda\left(u_*^I(\tau) + \theta + \sum_{j\neq c,j=1}^{n} u^j(\tau)\right) + \mathrm{e}^{-r\tau}\Delta(\tau)(-\tfrac{1}{2}(u^c)) \right\} = 0. \tag{9}$$

The terminal conditions from the definition of $\Phi^j(\tau, X)$ are given by[9]

$$\Phi^I(T, X) = \upsilon^{\omega}\Delta_c(T) - \upsilon^{\omega}\Delta(T) - F, \tag{10}$$

$$\Phi^c(T, X) = \upsilon^{\omega}\Delta_I(T) - \upsilon^{\omega}\Delta(T) - F. \tag{11}$$

The firm's equilibrium strategies take the form $u_*^I = u^I(\tau, X, \Phi_{X^I}^I(\tau, X))$ and $u_*^c = u^c(\tau, X, \Phi_{X^C}^c(\tau, X))$. Together with the terminal conditions in (10) and (11), we can write a system of suggestive solutions in the following form[10]

$$\Phi^I(\tau, X) = a^I(\tau)\Delta_c(\tau) + b^I(\tau)\Delta(\tau) - F, \tag{12}$$

[9] $\Phi_\tau^j(\tau, X)$, $\Phi_{X^I}^j(\tau, X)$ and $\Phi_{X^c}^j(\tau, X)$ denote the derivative of $\Phi^j(\tau, X)$ with respect to $\tau, X^I(\tau)$ and $X^c(\tau)$, respectively.

[10] $a^I(\tau)(B^I(\tau))$ and $a^c(\tau)(b^c(\tau))$ may not be identical since the incumbent I and challenger c are heterogeneous.

$$\Phi^c(\tau, X) = a^c(\tau)\Delta_I(\tau) + b^c(\tau)\Delta(\tau) - F. \tag{13}$$

Therefore, we have that $u^I_*(\tau, X) = -\lambda e^{\gamma\tau} b^I(\tau)$ and $u^c_*(\tau, X) = -\lambda e^{\gamma\tau} b^c(\tau)$ from (12) and (13). As in Reinganum (1982), we can prove the existence of a unique Nash equilibrium in Theorem 1.

Theorem 1. *Under Assumption* 1, *there exists a unique Nash equilibrium in the R&D competition.*

Proof. See Appendix A. □

Substituting the partial derivatives of the system of solutions in (12) and (13) with respect to X^I and X^c into the Bellman equations of (8) and (9) and using the terminal conditions of (10) and (11), we obtain a system of nonlinear differential equations and the boundary conditions:

$$\dot{a}^I(\tau) + 2n\lambda^2 e^{\gamma\tau} a^I(\tau) b^c(\tau) - \lambda^2 e^{\gamma\tau} \upsilon^\omega b^I(\tau) = 0, \tag{14}$$

$$a^I(T) = \upsilon^\omega,$$

$$\dot{a}^c(\tau) + \lambda^2 e^{\gamma\tau}(b^I(\tau) + b^c(\tau))a^c(\tau) - \lambda^2 e^{\gamma\tau}\upsilon^\omega b^I(\tau) + (n-1)\lambda^2 e^{\gamma\tau} b^c(\tau)(a^I(\tau)+1)$$
$$- \lambda\theta a^I(\tau) + \lambda\theta\upsilon^\omega = 0, \tag{15}$$

$$a^c(T) = \upsilon^\omega,$$

$$\dot{b}^I(\tau) + \tfrac{3}{2}\lambda^2 e^{\gamma\tau} b^I(\tau)^2 - \lambda\theta b^I(\tau) + 2n\lambda^2 e^{\gamma\tau} b^I(\tau) b^c(\tau)$$
$$+ \lambda^2 e^{\gamma\tau}(\upsilon^\omega - \upsilon^\ell) bc(\tau) + R e^{-\gamma\tau} = 0, \tag{16}$$

$$b^I(T) = -\upsilon^\omega,$$

$$\dot{b}^c(\tau) + (2n - \tfrac{1}{2})\lambda^2 e^{\gamma\tau} b^c(\tau)^2 - \lambda\theta b^c(\tau) + \lambda^2 e^{\gamma\tau}(2b^I(\tau) + (n-1)(\upsilon^\omega - \upsilon^\ell))b^c(\tau)$$
$$+ \lambda^2 e^{\gamma\tau}(\upsilon^\omega - \upsilon^\ell) b^I(\tau) - \lambda\theta(\upsilon^\omega - \upsilon^\ell) = 0, \tag{17}$$

$$b^c(T) = -\upsilon^\omega.$$

There exists a pair of general solutions: $a^I(\tau) = \upsilon^\omega$ and $a^c(\tau) = \upsilon^\omega$ for Eqs. (14) and (15). However, it is difficult to obtain a closed-form solution of $b^I(\tau)$ and $b^c(\tau)$ from the set of first-order nonlinear partial differential equations in (16) and (17).[11]

3. Numerical analysis and the firms' R&D strategies

In this section, we study how the $n+1$ firms' optimal investment strategies are affected by the marginal production experience (θ), the terminal value of being the

[11] $\dot{b}^I(\tau)$ and $\dot{b}^c(\tau)$ of (16) and (17) can be positive, or negative for $b^I(\tau) < 0$ and $b^c(\tau) < 0$.

winner (υ^{ω}), the terminal value of being the loser (υ^{ℓ}), the flow of monopoly profits (R) and the expected waiting time of an innovation ($1/\lambda$) as well as the rival's investment intensities in the model. In order to numerically solve the nonlinear differential equations of (16) and (17), we use the Runge–Kutta–Fehlberg method with the fourth-order and fifth-order Taylor series expansions, which is one of the most powerful methods for solving the nonlinear differential equations (see Fehlberg, 1969; Gerald and Wheatley, 1994). A discussion of the algorithm of this methodology is included in Appendix B.

In the following analyses, we assume that firms are convinced that the innovation is infeasible if they fail to succeed by $T = 4$.[12] We will also set the producer's discounted rate γ to be 0.1 and restrict υ^{ω} to be greater than υ^{ℓ}.

For the case of one incumbent and n identical challengers in the absence of production experience (i.e., $\theta = 0$), our numerical analyses confirm Reinganum's (1985) theoretical conclusions on how the equilibrium investment strategies of the incumbent and the challengers will change with the three factors: υ^{ω}, υ^{ℓ}, and R. That is, given the benchmark parameter $\theta = 0$, we show that: (i) the challenger invests more than the incumbent; and (ii) the incumbent's and challengers' investment intensities increase with υ^{ω} but decrease with υ^{ℓ} and R (see Appendix C, Figs. 10, 11 and 13). Furthermore, the numerical conclusions reveal that all competitors' investment decreases with the expected waiting time of an innovation ($1/\lambda$), which is also a confirmation of Malueg and Tsutsui's (1997) theoretical conclusion. In other words, firms become more aggressive and increase their R&D intensities as the expected waiting time of innovation ($1/\lambda$) decreases (Fig. 12 in Appendix C).

We then extend the Reinganum's (1982) framework to study the general situations when there are many challengers and when the incumbent enjoys the advantage of accumulated production experiences ($\theta > 0$). Since it is almost impossible to apply theoretical arguments to analyze how these paths change with the exogenous variables such as υ^{ω}, υ^{ℓ}, λ and R, our numerical analyses help us to obtain useful comparative static properties, which are not available in Reinganum (1982, 1983, 1985). By extending the benchmark case of $\theta = 0$ (Appendix C) to the general case of $\theta > 0$ and $n \geqslant 1$, we not only verify that the basic insights of the benchmark case still hold but also demonstrate that the influence of production experiences is significant.

We first examine how the incumbent's production experience (θ) affects the incumbent's and challengers' R&D investment rates. Figs. 1 and 2 illustrate these numerical analyses.

Result 1. For any given n ($n \geqslant 1$), (i) the incumbent always invests less than the challenger, i.e., $u_*^c - u_*^I > 0$ for any given value of production experiences (θ); and (ii) the incumbent's investment rate decreases with production experiences and the challengers' investment rates increase with the production experiences (θ).

[12]The variation in the values of T, θ, γ, υ^{ω}, υ^{l}, λ and R, as long as the equilibrium solution exists, does not affect the main conclusions of our numerical results.

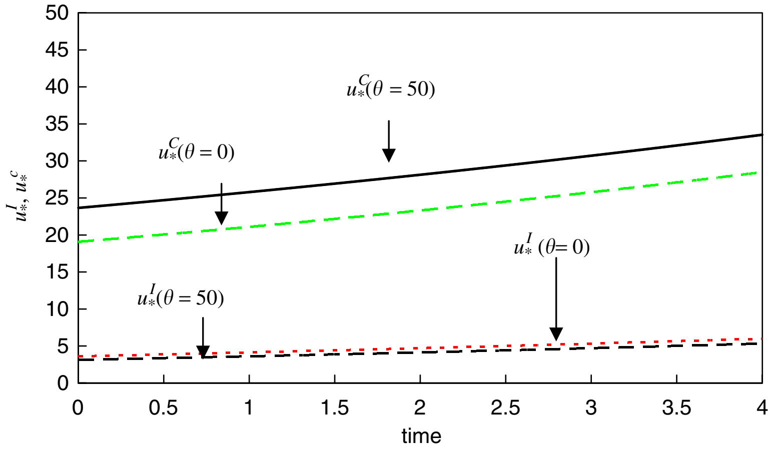

Fig. 1. The impact of θ for the first part for Result 1. Parameter specification: $\theta = 0, 50$, $\gamma = 0.1$, $\lambda = 0.3$, $\upsilon^{\omega} = 200$, $\upsilon^{\ell} = 50$, $R = 150$, $n = 5$ and $T = 4$.

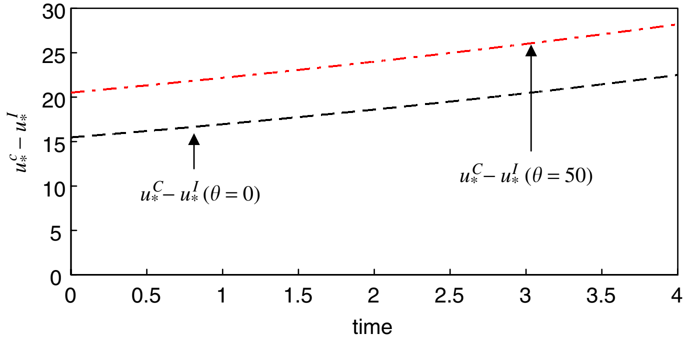

Fig. 2. The impact of θ the second part for Result 1. Parameter specification: $\theta = 0, 50$, $\gamma = 0.1$, $\lambda = 0.3$, $\upsilon^{\omega} = 200$, $\upsilon^{\ell} = 50$, $R = 150$, $n = 5$ and $T = 4$.

Result 1 shows that the difference between the challengers' and the incumbent's investment rates increases with production experiences (θ). The main reason is that the incumbent's instantaneous winning probability, $\exp\{-\lambda X(\tau)\}\lambda[u_I(\tau) + \theta]\,\mathrm{d}\tau$ as in Eq. (1a), increases with θ and the instantaneous probability for the challenger to lose, $\exp\{-\lambda X(\tau)\}[\lambda(u^I(\tau) + \theta + \sum_{j=1,j\neq c}^{n} u^j(\tau))]\,\mathrm{d}\tau$ as in Eq. (2b), also increases with θ (Fig. 1). Hence, the incumbent can remove some resources from the investment in the patent race to other productive department (that is to decrease investment) and still maintain its advantage in R&D competition as θ increases, while the challengers have to increase their investments to compete with a more experienced incumbent.

Our numerical results also reveal that the incumbent will accelerate investment rates for projects with higher winning rewards υ^{ω} or more optimistic outlook (i.e., a greater λ) and decelerate investment for higher R or υ^{ℓ}, confirming the existing results of Reinganum (1982, 1983, 1985). In addition, we explore further how the difference

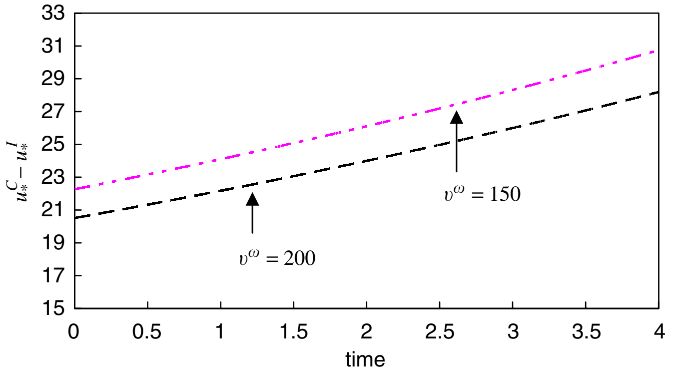

Fig. 3. The impact of υ^ω for Result 2. Parameter specification: $\upsilon^\omega = 150, 200$; $R = 150$; $\lambda = 0.5$; $\gamma = 0.1$; $\theta = 50$; $\upsilon^\ell = 50$; $n = 5$ and $T = 4$.

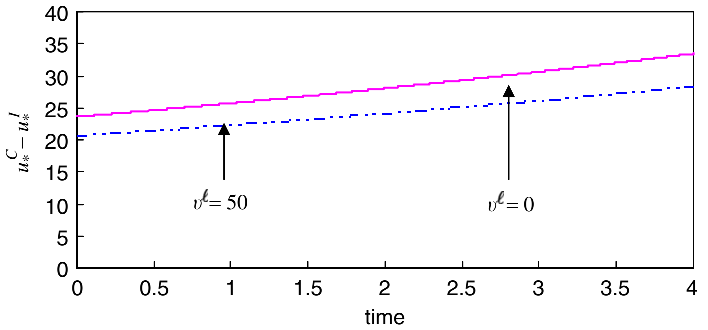

Fig. 4. The impact of υ^ℓ for Result 2. Parameter specification: $\upsilon^\ell = 0, 50$; $R = 150$; $\lambda = 0.5$; $\gamma = 0.1$; $\theta = 50$; $\upsilon^\omega = 200$; $n = 5$ and $T = 4$.

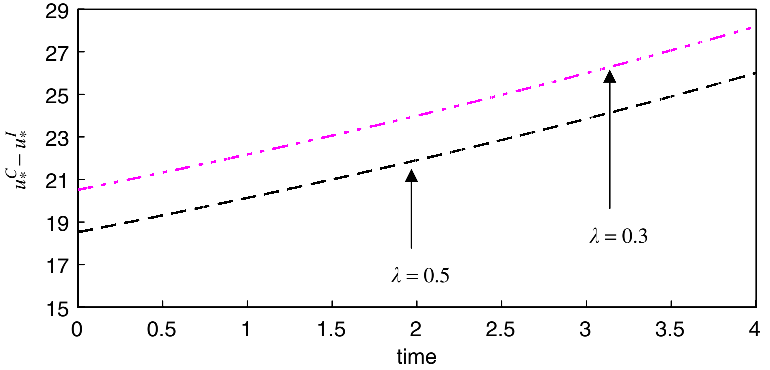

Fig. 5. The impact of λ for Result 2. Parameter specification: $\lambda = 0.3, 0.5$, $\gamma = 0.1$, $\theta = 50$, $\upsilon^\omega = 200$, $\upsilon^\ell = 50$, $R = 150$, $n = 5$ and $T = 4$.

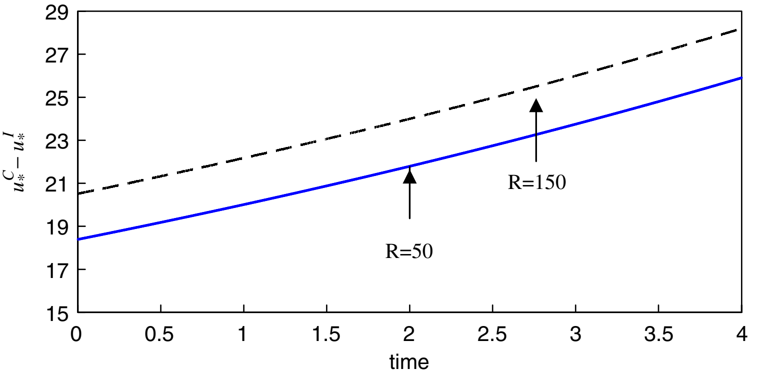

Fig. 6. The impact of R for Result 2. Parameter specification: $R = 50, 150$; $\lambda = 0.5$; $\gamma = 0.1$; $\theta = 50$; $\upsilon^{\omega} = 200$; $\upsilon^{\ell} = 50$; $n = 5$ and $T = 4$.

between the challengers' and incumbent's investment rates is affected by these factors. Figs. 3–6 illustrate that the difference between the challengers' and the incumbent's investment rates ($u_*^c - u_*^I$) decreases with υ^{ω}, λ and υ^{ℓ} but increases with R. Result 2 provides useful comparative static properties as an extension of Reinganum's (1982, 1983, 1985) theoretical conclusions.

Result 2. For any given $\theta \geqslant 0$ and $n \geqslant 1$, the difference between the challengers' and the incumbent's investment rates ($u_*^c - u_*^I$) decreases with the value of being a winner (υ^{ω}) and the terminal value of being a loser (υ^{ℓ}) but increases with the expected waiting time of an innovation ($1/\lambda$) and the flow of monopoly profits (R).

In addition, we can obtain further characterization of the dependence of the likelihood of the incumbent keeping winning on the variables of its production experiences, the number of competitors, the winner's rewards, the loser's payoff, and the expected innovative time of research project, etc. We obtain an analytical result in Theorem 2.

Theorem 2. *The likelihood of persistent leadership is positive. It increases with the marginal production experience* (θ) *but decreases with the number of challengers* (n).

Proof. Since $\exp\{-\lambda X(\tau)\}\lambda[u^I(\tau) + \theta]\,\mathrm{d}\tau$ of (1a) is the incumbent's instantaneous winning probability, and the instantaneous probability for the incumbent to lose at time τ is $\exp\{-\lambda X(\tau)\}\lambda \sum_{c=1}^{n} u^c(\tau)\,\mathrm{d}\tau$ of (2a), for a sufficiently large value of θ, we must have $\theta > nu_*^c - u_*^I$. In other words, there is a positive probability of persistent leadership in the R&D race if $\theta > nu_*^c - u_*^I$, and this probability increases with θ, and declines with n. $\square$

From Result 2, for a given value θ and n, the difference between challenger's and incumbent's investment rates decrease with υ^{ω} and υ^{ℓ}, but increase with the value of $1/\lambda$ and R. Hence, by Theorem 2, the likelihood of persistent leadership (or

$\theta > nu_*^C - u_*^I$) increases if υ^ω or υ^ℓ becomes sufficiently large, or if $1/\lambda$ or R becomes sufficiently small. The following result shows that the variations of the incumbent's and the challengers' equilibrium investment strategies with these factors: (υ^ω, υ^ℓ, λ and R) and are the same as Reinganum's (1985) conclusions ($\theta \ = 0$) no matter whether the incumbent's production experiences are positive (the case of $\theta > 0$) or zero ($\theta = 0$). Thus, from Result 2, we obtain that the likelihood of persistent leadership increases in υ^ω and υ^ℓ but decreases with $1/\lambda$ and R. Note that the likelihood of persistent leadership is just the opposite of that of destructive innovation.

Result 3. For any given θ ($\theta \geqslant 0$) and n ($n \geqslant 1$), the likelihood of persistent leadership increases with the value of being a winner (υ^ω) and the value of being a loser (υ^ℓ) but decreases with the expected waiting time of an innovation ($1/\lambda$) and the flow of monopoly profit (R).

Similar with the impact of the reward of being a loser on the firms' investment strategies (Results 2 and 3), increasing R&D competition decelerates the incumbent's investment relative to those of its rivals as in Result 4.

Since the incumbent's instantaneous probability of losing the race, $\exp\{-\lambda X(\tau)\}\lambda \sum_{c=1}^{n} u^c(\tau)\,\mathrm{d}\tau$, and the challengers' instantaneous probabilities of losing the race, $\exp\{-\lambda X(\tau)\}[\lambda(u^I(\tau) + \theta + \sum_{j=1, j\neq c}^{n} u^j(\tau))]\,\mathrm{d}\tau$, both increase with the number of competitors, their investment intensities decrease in competition (Fig. 7). But, as shown in Fig. 8, the difference of investment rates between the incumbent and challengers increase with the number of competitors. Therefore, the likelihood of persistent leadership can decrease with the number of challengers (see Fig. 9).

Result 4. For any given θ ($\theta \geqslant 0$), (i) the incumbent's and challengers' investment intensities decrease with the number of challengers; (ii) the difference between the

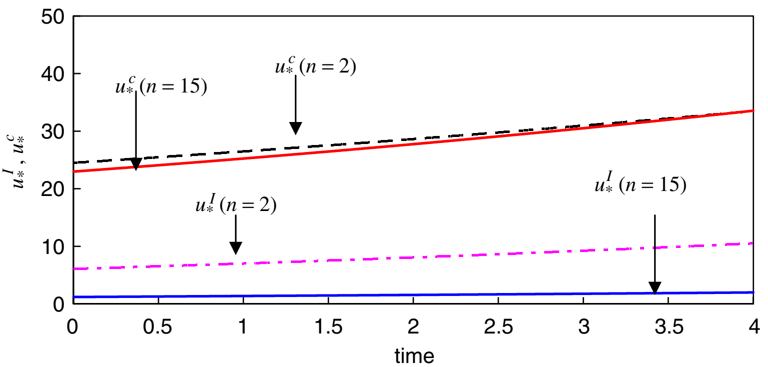

Fig. 7. The impact of n on u_*^I and u_*^c for Result 4. Parameter specification: $n = 2, 15$, $\gamma = 0.1$, $\lambda = 0.3$, $\upsilon^\omega = 200$, $\upsilon^\ell = 50$, $R = 150$, $\theta = 50$ and $T = 4$.

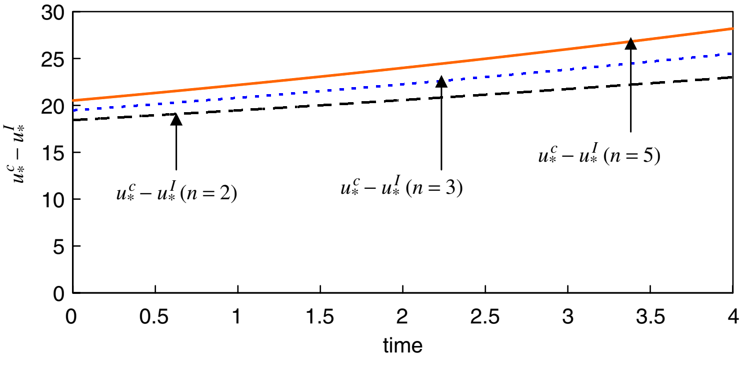

Fig. 8. The impact of n on $u_*^c - u_*^I$ for Result 4. Parameter specification: $n = 2, 3, 5$, $\gamma = 0.1$, $\lambda = 0.3$, $\upsilon^{\omega} = 200$, $\upsilon^{\ell} = 50$, $R = 150$, $\theta = 50$ and $T = 4$.

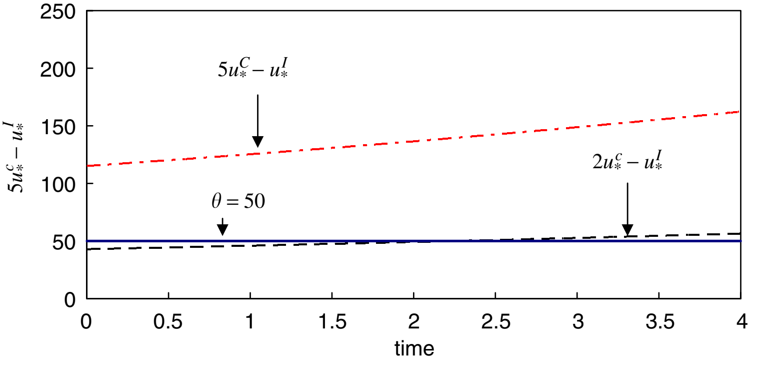

Fig. 9. The impact of n to the likelihood of persistent leadership for Result 4. Parameter specification: $n = 2, 5$, $\gamma = 0.1$, $\lambda = 0.3$, $\upsilon^{\omega} = 200$, $\upsilon^{\ell} = 50$, $R = 150$, $\theta = 50$ and $T = 4$.

incumbent's and challengers' investment intensities increases with the number of challengers; and (iii) the likelihood of persistent leadership decreases with the number of challengers.

Furthermore, if the production experiences play an important role (i.e., $\theta > nu_*^c - u_*^I$) as in Theorem 2, the persistent leadership can become even more likely given other things being equal. Our numerical analysis with Fig. 9 reveals that the persistent leadership can take place under quite general situations so long as the incumbent's production experiences is sufficiently large to satisfy the condition $\theta > nu_*^c - u_*^I$. For instance, with parameter values specified in Fig. 9, the persistent leadership can occur when the number of challengers equals to 2 and the accumulated production experience is $\theta\tau$ with $\tau \in [0, 2.234]$ such that $\theta = 50 > 2u_*^c - u_*^I$ is satisfied.

Table 1
The determining factors of the firms' investment rates and the likelihood of persistent leadership[a]

Investment rates/Factors	V^{ω}	V^{ℓ}	R	λ	θ	n
u_*^I	+	−	−	+	−	−
u_*^c	+	−	−	+	+	−
$u_*^c - u_*^I > 0$	−	−	+	−	+	+
Likelihood of persistent leadership	+	+	−	+	+	−

[a]The symbol + indicates that the relationship is positive.

4. Concluding remarks

We summarize the similarities and differences among the results of this paper and those of Reinganum (1983, 1985) and Malueg and Tsutsui (1997) in Table 1. First, for the influence of production experiences, we extend the theoretical analyses of Reinganum's (1983) and show that the incumbent can keep its persistent leadership even when its optimal investment is less than those of the challengers. The realities of CPU competition mentioned before provided a good example. On the other hand, the persistent leadership is less likely to occur in an industry with low-tech entry barrier and hence with many challengers, as in the OPC industry mentioned before. This has been demonstrated in Theorem 2 and recorded in the last two cells of the last row of Table 1. This result constitutes a part of the new contributions of this paper.

Secondly, by applying numerical method, we show for the first time how the incumbent's and the challengers' investment rates change with production experiences. This has been our Result 1 and recorded in the fifth column of Table 1.

Thirdly, in addition to confirming how the perspective investment rates of the incumbent and the challengers are influenced by factors, V^{ω}, V^{ℓ} and R (as in Reinganum, 1985) and λ (as in Malueg and Tsutsui, 1997), we also analyze how the difference between the challengers' and the incumbent's investment rates changes with these determining factors (Result 2). This has been recorded in the first three rows of Table 1.

Fourthly, we also analyze the dependence of the likelihood of persistent leadership on the determining factors, V^{ω}, V^{ℓ}, λ and R (Result 3). Our results, as recorded in the last row of Table 1, confirm and extend those of Reinganum (1985) and Malueg and Tsutsui (1997).

Lastly, we show how the increase in the number of challengers affects the investment rates of participants and hence the difference between their investment rates (Result 4). Our results extend the existing analysis (see Malueg and Tsutsui, 1997) to the case with production experiences and are recorded in the last column of Table 1.

The realities of the CPU and EPROMs industries unveil that production experiences play an important role in the patent race of new generation products. Hence, it seems unreasonable to assume that all participants are identical in sharing prior resources before participating in the patent race, as in Reinganum (1982). The major feature of our model is that the incumbent enjoys an advantage of learning from production experiences and receives monopoly profit from the post-innovation products in the R&D competition, while each challenger has to pay a lump sum entry cost and recurrent flow costs under the R&D competition. Our model can also provide useful insights to industries other than the examples mentioned in the paper, such as races in medicine research. Consequently, we can say that our results provide a quite complete analysis of the R&D competition with production experiences and other exogenous influences.

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Appendix A. The proof of Theorem 1

We first prove the existence of a Nash equilibrium. From the Theorems of Friedman (1971), we know that the optimal strategies must satisfy a system of Bellman equations (see the discussion in Section 2.2). Since the left-hand sides of the Bellman equations in (8) and (9) are strictly concave in u^I and u^c, respectively, the necessary conditions are also sufficient for optimal solutions. We will focus on the interior solutions of these Bellman equations, which are given by

$$u^I_* = e^{r\tau}e^{\lambda X(\tau)}\Phi^I_{X^I}(\tau, X), \tag{A.1}$$

$$u^c_* = e^{r\tau}e^{\lambda X(\tau)}\Phi^c_{X^c}(\tau, X). \tag{A.2}$$

The boundary solutions include: $u^j_* = 0$ as $\Phi^j_{x^j}(\tau, X) < 0$, and $u^j_* = K$ as $\Phi^j_{x^j}(\tau, X) > 0$, $j = I, 1, 2, \ldots, n$. The solutions of (A.1) and (A.2) can be substituted into the Bellman equations of (8) and (9). Therefore, if we can solve the value functions (6) and (7) with the Hamilton–Jacobi equations, (8) and (9), then the firms' Nash equilibrium investment strategies exist as $u^I_* = u^I(\tau, X, \Phi^I_{X^I}(\tau, X))$ and $u^c_* = u^c(\tau, X, \Phi^c_{X^C}(\tau, X))$.

Note that firms' value functions obtained in Eqs. (A.1) and (A.2) with the terminal conditions of Eqs. (10) and (11) are continuously differentiable in X. Since $u^I_* = u^I(\tau, X, \Phi^I_{X^I}(\tau, X))$ and $u^c_* = u^c(\tau, X, \Phi^c_{X^c}(\tau, X))$, $c = 1, 2, \ldots, n$ satisfy the hypothesis of the sufficiency Theorem 1 (Stalford and Leitman, 1973), the $n+1$-firms' strategy tuple $(u^I_*, u^1_*, u^2_*, \ldots, u^n_*)$ is a Nash equilibrium for $(u^I_*, u^1_*, u^2_*, \ldots, u^n_*) \in \Omega^I \times \Omega^1 \times \Omega^2 \times \cdots \times \Omega^n$.

Once we prove this, then we can show the existence of an unique equilibrium by Theorem 18.1 of Bernstein (1950).

Recall that $u^I_*(\tau, X) = -\lambda e^{\gamma\tau} b^I(\tau)$ and $u^c_*(\tau, X) = -\lambda e^{\gamma\tau} b^c(\tau)$ from (12) and (13). Since the strategy space satisfies continuity, boundary and the Lipschitz conditions (Eq. (5)) in X by Assumption 1, and $b^I(\tau)$ and $b^c(\tau)$ are also continuously differentiable in X, then there exists an unique Nash equilibrium, $(u_*^I, u_*^1, u_*^2, \ldots, u_*^n)$ in $\Omega^I \times \Omega^1 \times \Omega^2 \times \cdots \times \Omega^n$ such that the firms' value functions are of class C^∞ in $(\tau, X, \Phi_{X^c}^c(\tau, X))$, and the terminal conditions are of class C^∞ in X, respectively (Bernstein's (1950) Theorem 18.1).

Therefore, we verify that each firm's value function as characterized by a pair of Eqs. (6) and (7) is of class C^∞ in $(\tau, X, \Phi_{X^c}^c(\tau, X))$ and the terminal conditions of Eqs. (10) and (11) are of class C^∞ in X after substituting $u_*^I = \mathrm{e}^{r\tau}\mathrm{e}^{\lambda X(\tau)}\Phi_{X^I}^I(\tau, X)$ and $u_*^c = \mathrm{e}^{r\tau}\mathrm{e}^{\lambda X(\tau)}\Phi_{X^c}^c(\tau, X)$ into the Bellman equations of (8) and (9). There exists only one solution to (6), (10) and (7), (11). Consequently, the $n+1$ tuple $(u_*^I, u_*^1, u_*^2, \ldots, u_*^n)$ is the unique Nash equilibrium in $\Omega^I \times \Omega^1 \times \Omega^2 \times \cdots \times \Omega^n$. □

Appendix B. An algorithm for the Runge–Kutta–Fehlberg method

Because the numerical simulation processes of both (16) and (17) are similar, we use (12) to introduce the methodology and its regularity conditions. For developing the relationship between $b^A(\tau^*)$ ($b^I(\tau^*)$ of Eq. (12)) and τ^* by the Taylor series (see Judd, 1998), we find the coefficients of the Taylor series where $b^A(\tau^*)$ expands around the point $\tau^* = T$:

$$b^A(\tau^*) = b^A(T) + b'^A(T)h + \frac{b''^A(T)}{2!}h^2 + \frac{b'''^A(T)}{3!}h^3 + \cdots, \quad \tau^* - T = h.$$

The Taylor-series method and the (modified) Euler method are not suitable for our first-order nonlinear differential equations. We find that it is far better to use a more efficient method such as the following (modified) Runge–Kutta methods rather than to use many re-corrections in the modified Euler method. The Runge–Kutta methods are the equivalent of approximating the exact solution by matching the first n terms of the Taylor-series expansion. For example, in a second-order Runge–Kutta method (see Hubbard and West, 1990), we let the increment of b_{n+1}^A be a weighted average of two estimates of the increment, k_1 and k_2, where

$$k_1 = hf(\tau^*, b_n^A) \quad \text{and} \quad k_2 = hf(\tau^* + \alpha h, b_n^A + \beta k_1) \tag{B.1}$$

for $\mathrm{d}b^A/\mathrm{d}\tau^* = f(b^A, \tau^*)$. Eq. (B.1) suggests that the Runge–Kutta methods use the Euler estimate as the first estimate of Δb^A (i.e. k_1); the second estimate is created by the increments of τ^* and b^A with the fractions α and β of h and of the earlier estimate k_1. The second step is to devise a scheme of choosing the four parameters, a, b, α and β. We find that the modified Euler method is a special case of a second-order Runge–Kutta method. There exist three equations to be satisfied by four unknowns in a second-order Runge–Kutta method.

$$b_{n+1}^A = b_n^A + hf(b_n^A, \tau^*) + h^2(\tfrac{1}{2}f_{\tau^*} + \tfrac{1}{2}f_{b^A}f), \tag{B.2}$$

$$b_{n+1}^A = b_n^A + ahf(b_n^A, \tau^*) + bhf[\tau^* + \alpha h, b_n^A + \beta hf(b_n^A, \tau^*)], \tag{B.3}$$

$$b_{n+1}^A = b_n^A + (a+b)hf(b_n^A, \tau^*) + h^2(\alpha bf_{\tau^*} + \beta bf_{b_n^A}f). \tag{B.4}$$

In general, the fourth-order Runge–Kutta methods (see Judd, 1998) are most widely used. However, the higher order Runge–Kutta method becomes more complicated and expensive in re-computing the values. One of better approaches is using two Runge–Kutta methods, though of different orders of errors, to move from (b_n^A, τ_n^*) to $(b_{n+1}^A, \tau_{n+1}^*)$. The Runge–Kutta–Fehlberg method was introduced by Fehlberg to compare the methods of two different orders to increase the efficiency of the Runge–Kutta methods (see Fehlberg, 1969; Gerald and Wheatley, 1994). Since this method requires far less function evaluations than the Runge–Kutta methods and provides a mechanism to adjust the step size h depending on the value of the estimated error, it has become a very useful method.

The Runge–Kutta–Fehlberg method contains the following three steps. The first step is to compute two Runge–Kutta estimates for b_{n+1}^A, though of different orders (fourth- and fifth-order Runge–Kutta formulas) of the errors. Secondly, we compare the two estimates of b_{n+1}^A created by the fourth- and fifth-order Runge–Kutta formulas, respectively. The final step is the adjustment of the step size h depending on the value of the estimated error as required. As with the above algorithm for the Runge–Kutta–Fehlberg method, we only need six function evaluations. In addition, instead of comparing estimates of b_{n+1}^A for the step sizes h and $h/2$, we can choose a reasonable value of h from the value of estimated error. Thus, we choose to apply the following algorithm of the Runge–Kutta–Fehlberg method to our numerical problems.

$$\begin{aligned}
k_1 &= hf(x_n, y_n),\\
k_2 &= hf(\tau^* + \tfrac{1}{4}h, b_n^A + \tfrac{1}{4}k_1),\\
k_3 &= hf(\tau^* + \tfrac{3}{4}h, b_n^A + \tfrac{3}{32}k_1 + \tfrac{9}{32}k_2),\\
k_4 &= hf\left(\tau^* + \frac{12h}{13}, b_n^A + \frac{1932k_1}{2197} - \frac{7200k_2}{2197} + \frac{7296k_3}{2197}\right),\\
k_5 &= hf\left(\tau^* + h, b_n^A + \frac{439k_1}{216} - 8k_2 + \frac{3680k_3}{513} - \frac{845k_4}{4104}\right),\\
k_6 &= hf\left(\tau^* + \frac{h}{2}, b_n^A - \frac{8k_1}{27} + 2k_2 - \frac{3544k_3}{2565} + \frac{1859k_4}{4104} - \frac{11k_5}{40}\right),\\
\hat{b}_{n+1}^A &= b_n^A + \left(\frac{25k_1}{216} + \frac{1408k_3}{2565} + \frac{2197k_4}{4104} - \frac{k_5}{5}\right), \text{ with global error O}(h^4),\\
b_{n+1}^A &= b_n^A + \left(\frac{16k_1}{135} + \frac{6656k_3}{12825} + \frac{28561k_4}{56430} - \frac{9k_5}{50} + \frac{2k_6}{55}\right),\\
&\text{with global error O}(h^5),\\
\text{Error, } E &\cong \frac{k_1}{360} - \frac{128k_3}{4275} - \frac{2197k_4}{75240} + \frac{k_5}{50} + \frac{2k_6}{55}.
\end{aligned} \tag{B.5}$$

Appendix C. The benchmark parameter case ($\theta = 0$)

In this appendix, we consider the benchmark case of $\theta = 0$ (the absence of production experiences). We show that the challenger invests more than the incumbent and their investment rates increase with υ^{ω} but decrease with υ^{ℓ} and R (Figs. 10–13), confirming the results of Reinganum (1985). In addition, their investment rates decrease with the expected waiting time of an innovation ($1/\lambda$), supporting the conclusion of Malueg and Tsutsui (1997).

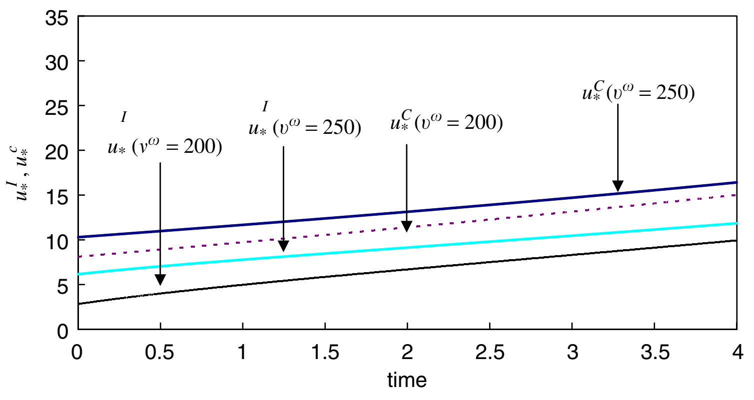

Fig. 10. The influence of υ^{ω} on u_*^I and u_*^c. Parameter specification: $\upsilon^{\omega} = 200, 250$; $\gamma = 0.1$, $\lambda = 0.2$, $\theta = 0.0$, $\upsilon^{\ell} = 50$, $R = 150$, $n = 1$ and $T = 4$.

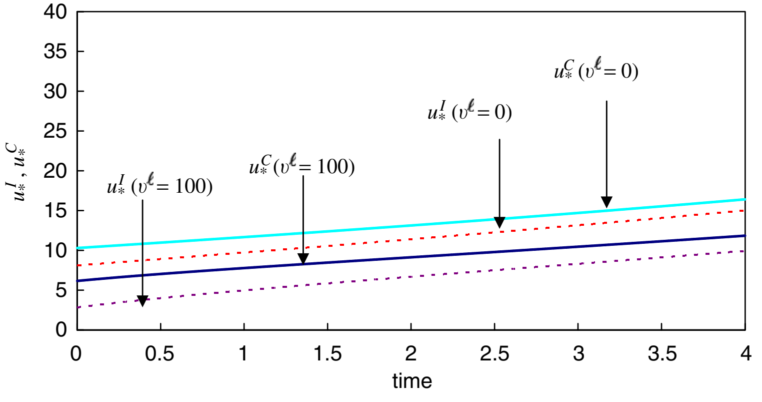

Fig. 11. The influence of υ^{ℓ} on u_*^I and u_*^c. Parameter specification: $\upsilon^{\ell} = 0, 50$; $R = 150$; $\lambda = 0.2$; $\gamma = 0.1$; $\theta = 0.0$; $\upsilon^{\omega} = 200$; $n = 1$ and $T = 4$.

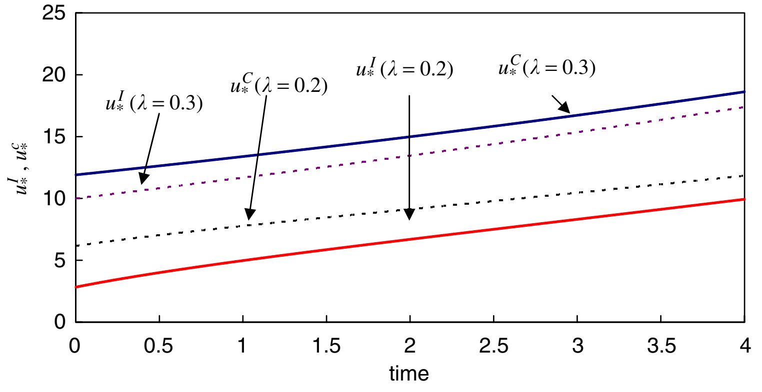

Fig. 12. The influence of λ on u_*^I and u_*^c. Parameter specification: $\lambda = 0.2, 0.3$, $\gamma = 0.1$, $\theta = 0.0$, $\upsilon^{\omega} = 200$, $\upsilon^{\ell} = 50$, $R = 150$; $n = 1$ and $T = 4$.

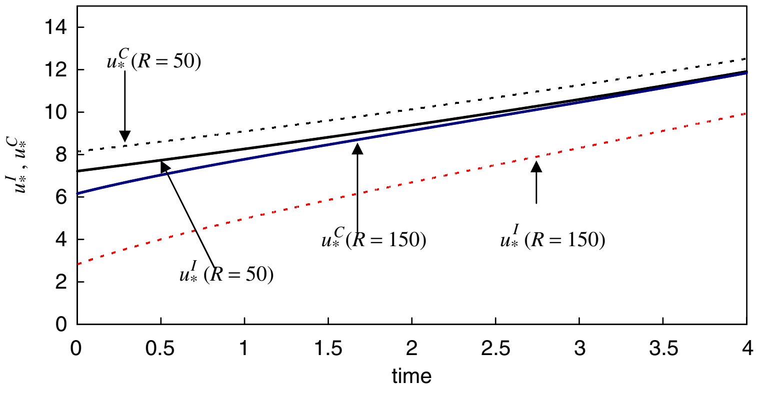

Fig. 13. The influence of R on u_*^I and u_*^c. Parameter specification: $R = 50, 150$; $\lambda = 0.2$; $\gamma = 0.1$; $\theta = 0.0$; $\upsilon^{\omega} = 200$; $\upsilon^{\ell} = 50$; $n = 1$ and $T = 4$.

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