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Do High-Frequency Data Improve Multivariate Volatility Forecasting for Investors with Different Investment Horizons?

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Abstract This study investigates the role of high-frequency data in multivariate volatility forecasting for investors with different investment horizons. We use six multivariate volatility models with high-frequency and low-frequency data for a sample of 10 Dow Jones stocks and evaluate the performance of forecast volatility based on both statistical and economic methods. In our statistical evaluation, we find that high-frequency data significantly enhance forecast accuracy over the daily horizon, but this improvement is dampened when longer horizons are used. In our economic evaluation, we find that high-frequency data cannot improve all economic benefits under the short and long horizons. The economic benefits of using high-frequency data depend on the evaluation framework.

Key Words: Realized Volatility, Covariance Matrix Forecasting, Investment Horizon, Statistical and Economic Evaluation

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1. Introduction

Modeling and forecasting covariance matrices play an important role in many areas in finance, such as derivative pricing, portfolio selection, and risk management. One popular approach is to use multivariate generalized autoregressive heteroscedasticity (MGARCH)-type models. Bollerslev (1986) first proposed the univariate GARCH model. After that, Engle and Kroner (1995) and Engle (2002a) introduced the Baba–Engle–Kraft–Kroner (BEKK) and dynamic conditional correlation (DCC) models, which have become two of the most widely used models for conditional covariance matrix forecasting. These MGARCH models only use daily information to forecast the conditional covariance matrices and characterize the return vector. However, the daily squared return is the noise proxy for measuring the true conditional covariance matrix. Hence, researchers have constructed other estimators to measure true volatility and incorporate them into volatility modeling.

Researchers need high-frequency data to measure true volatility, which are now easily available because of advances in technology. Andersen and Bollerslev (1998) introduced realized volatility based on high-frequency data. Barndorff-Nielsen and Shephard (2004) proposed the realized covariance matrix, a consistent proxy in the absence of market microstructure. Barndorff-Nielsen et al. (2011) provided an alternative realized measure, the multivariate realized kernel, which takes full advantage of high-frequency data and overcomes the problem of market microstructure.

Realized volatility can be quickly incorporated into volatility modeling. A natural step is to extend the GARCH-type model based on realized measures. The univariate GARCH-X model of Engle (2002b) was one of the first GARCH-type models to utilize realized volatility as an exogenous variable. Other univariate GARCH-type models soon followed, such as the MEM model (Engle and Gallo, 2006), the high frequency-based volatility (HEAVY) model (Shephard and Sheppard, 2010), and the Realized GARCH model (Hansen et al., 2012). To incorporate realized measures into realized covariance matrix forecasting, researchers have introduced MGARCH models with realized volatility, including the multivariate HEAVY model (Noureldin et al., 2011), the Realized Wishart GARCH (RWG) model (Gorgi et al., 2019), and the multivariate Realized GARCH (MRG) model (Archakov et al., 2020). In addition to GARCH-type models, the multivariate Realized Exponentially Weighted Moving Average (REWMA) model (Fleming et al., 2003) and the multivariate Heterogeneity Autoregressive (HAR) model (Chiriac and Voev, 2011) have become popular in the field of conditional covariance matrix forecasting.

Many researchers have focused on evaluating the role of high-frequency data in volatility forecasting and have reached a consensus that models that use realized measures outperform those that use only daily information in terms of volatility forecasting. Martens (2001) and Andersen et al. (2003) indicated that GARCH models using intra-day high-frequency data perform better than standard GARCH models in daily volatility forecasting. Maheu and McCurdy (2011) demonstrated that realized volatility is beneficial for fitting return distributions. Amendola et al. (2020) concluded that the combination of low- and high-frequency multivariate covariance forecasts is more accurate than covariance forecasts using daily information.

However, most of these comparisons have been based on short-term daily rebalancing frequency. There has been limited evidence of the role of high-frequency data in volatility forecasting over long horizons. Although Lyócsa et al. (2021) paid attention to the performance difference between high-frequency and low-frequency models over long horizons, they only focused on the univariate volatility model. Multi-step forecasts of conditional covariance matrices have broad applications in the area of finance. When constructing trading strategies, the position rebalancing frequency is typically maintained at a longer horizon due to transaction costs and trading restrictions, so investors usually choose to forecast conditional covariances over a longer horizon. Furthermore, volatility has a long memory property (Engle et al., 2013), so using realized volatility measures to forecast multi-period volatility may distort their stability. Thus, evaluating the role of realized volatility in multi-period volatility forecasting is important, as it guides investors on the necessity of using high-frequency data. To fill this gap in the literature, this study brings together the literature on multivariate volatility modeling and portfolio construction and focuses on the necessity of applying advanced multivariate volatility models using high-frequency data to invest in different horizons.

To achieve this goal, we apply different methods to solve the estimation problems of multivariate volatility models (Engle, 2009). We use composite likelihood to estimate large covariance matrices to eliminate the problems created by directly estimating a large dimension covariance matrix. According to Engle (2009) and Pakel et al. (2021), estimators are computationally costly and biased when directly estimating models with a large number of assets. Thus, researchers have used various methods to solve this problem (Engle, 2009; Engle and Kelly, 2012; Engle et al., 2019; Pakel et al., 2021; De Nard et al., 2022). Engle (2009) proposed the MacGyver method to forecast a covariance matrix by combining individual volatility forecasts and all pairwise correlations. The factor model of Engle and Kelly (2009) and Archakov et al. (2020) provides a model structure to reduce dimensions by dividing a covariance matrix into several blocks. Engle et al. (2019) proposed the DCC with nonlinear shrinkage estimators (DCC-NL) model with nonlinear shrinkage estimation to overcome dimensionality, and De Nard et al. (2022) further used open/high/low/close prices instead of simple daily returns to estimate the DCC-NL model, leading to better performance by large dynamic covariance matrices. Pakel et al. (2021) introduced composite likelihood by converting a $n \times n$ problem into several 2×2 problems. Composite likelihood provides us with consistent estimators and ensures the positive definitiveness of the covariance matrix without imposing any restrictions.

We consider both statistical and economic evaluations of different types of volatility models. The statistical evaluation is used to compare the forecast accuracy of different models, which is measured by robust loss functions[2]. We use the mean squared error (MSE) and QLIKE losses as robust loss functions (Patton and Sheppard, 2009; Laurent et al., 2013). We use the model confidence set (MCS) of Hansen et al. (2011) for the comparison method. Furthermore, we focus on economic evaluation methods, which are based on the out-of-sample performance of optimal portfolios. We use widely used economic losses (Fleming et al., 2003; DeMiguel et al., 2009; Chiriac and Voev, 2011; Callot et al., 2017; Bollerslev et al., 2018; Golosnoy and Gribisch, 2022; Grønborg et al., 2022) and compare the performance of the models based on the MCS approach.

In this study, we use six multivariate volatility models with high- and low-frequency data for a sample of 10 Dow Jones stocks to compare the forecast accuracy and portfolio selection performance of the models and examine how this pattern changes under daily, weekly, and monthly horizons. We demonstrate that high-frequency data have significantly better forecast accuracy than daily rebalancing frequency, but this improvement is dampened when we use longer (weekly and monthly) horizons. In addition, investors may not obtain significant economic benefits from high-frequency data, depending on which economic losses they pay more attention to. Compared with daily information, high-frequency data significantly reduce portfolio variances but cannot increase

[2] For the definition of "robust loss function," please see Patton (2011).

investor utilities and refine portfolio structures. The results reveal that asset allocation based on high-frequency data does not outperform that based on low-frequency data over long investment horizons.

This study contributes to the literature in the following three aspects. First, we investigate the role of high-frequency data in covariance matrix forecasting over different horizons, which has not been fully discussed in the literature. Second, we consider a broad class of models. We are the first to use RWG and MRG models to analyze the role of high-frequency data in covariance forecasts and portfolio performance under different rebalancing frequencies, making our results robust. Third, we apply appropriate estimation methods and comprehensive evaluation methods to conduct our empirical analysis.

The remainder of this paper is organized as follows. Section 2 discusses the three types of volatility models that we use in our empirical analysis. Section 3 presents additional estimation and forecasting details. Section 4 outlines the data and summary statistics. Section 5 reports the out-of-sample forecasting procedure, forecast evaluation methods, and the corresponding empirical results and Section 6 concludes.

2. Model

First, we present the three types of multivariate volatility models that we use in this study. As we are interested in a model's forecasting error and its implications for portfolio construction, we compare the performance of six models in terms of forecasting ability and portfolio selection. Section 2.1 introduces the notations and definitions used. Sections 2.2, 2.3, and 2.4 discuss the three types of models.

These candidate models can be divided into three groups where models in the first group use only daily information to forecast the daily covariance matrix and characterize the daily return distribution, models in the second group rely on intra-day realized volatility and daily data to describe the dynamics of the daily covariance matrix and daily return distribution, and models in the third group use both intra-day and daily data to forecast only the daily covariance matrix.

2.1 Notations and Definitions

2.1.1 Notations

Let r_t denote an n-dimensional vector of asset returns in period *t*. In this study, a period represents a trading day. The conditional mean and conditional covariance matrix of asset returns are denoted by

$$\mu_t = E(r_t|\mathcal{F}_{t-1})$$
$$H_t = Var(r_t|\mathcal{F}_{t-1})$$

where $\mathcal{F}_t$ is the natural filtration for $\{r_t,\ RV_t\}$. Here, RV_t represents the realized measure of H_t.

We decompose the conditional covariance matrix into conditional variance and correlation,

$$H_t = \Lambda_{h_t}^{1/2} C_t \Lambda_{h_t}^{1/2}$$

where $\Lambda_{h_t} = diag(h_t) = diag(h_{1,t}, \dots, h_{n,t})$ with $h_{i,t} = [H_t]_{ii}$, meaning that $h_{i,t}$ is the conditional variance of asset *i*'s return $r_{i,t}$, and C_t is the conditional correlation matrix.

We also use realized measures to model conditional variance and correlation. $x_t = diag(RV_t)$ denotes the realized measure of conditional variance h_t, and the corresponding realized measure of the conditional correlation matrix C_t is denoted by

$$Y_t = \Lambda_{x_t}^{-1/2} RV_t \Lambda_{x_t}^{1/2}$$

where $\Lambda_{x_t} = diag(x_t) = diag(x_{1,t}, \ldots, x_{n,t})$.

We let $N^* = n \times (n+1)/2$, $N = n \times (n-1)/2$, and l_n denote the $n \times 1$ vector with all elements equal to 1.

2.1.2 Realized Volatility

Realized volatility is a key element in volatility forecasting and portfolio selection. We use the realized covariance matrix as the realized measure of the conditional covariance matrix. Assume that we observe *k* uniformly spaced intra-day asset returns. We define the realized covariance matrix as the sum of the outer product of these intra-day returns:

$$RC_t = RV_t = \sum_{j=1}^{k} r_t^{(j)} r_t^{(j)\prime}$$

where $r_t^{(j)}$ denotes the *j*-th observation of the asset return vector on trading day *t*. The realized covariance matrix is computed using the intra-day return data as described in Section 4.

According to Barndorff-Nielsen and Shephard (2004), in the absence of market microstructure, RC_t is a consistent estimator as $k \to \infty$. Although RC_t is biased in the presence of market microstructure, we choose to sample sparsely and apply subsampling. One can also choose other estimators, such as the multivariate realized kernel of Barndorff-Nielsen et al. (2011), to overcome the problem of market microstructure.

2.2 Modeling Using Daily Data

2.2.1 DCC-GARCH Model

The DCC model of Engle (2002a) is a widely used multivariate GARCH model that uses only daily low-frequency information, is highly parsimonious, and ensures the positive definiteness of the covariance matrix. The DCC model describes the dynamics of conditional variance and correlation separately as follows:

$$r_t = \Lambda_{h_t}^{1/2} z_t, \qquad z_t \sim N(0, C_t) \tag{1}$$

$$h_t = \omega + \beta h_{t-1} + \alpha r_t \circ r_t \tag{2}$$

$$C_t = Q_t^{*-1/2} Q_t Q_t^{*-1/2} \tag{3}$$

$$Q_t = \bar{Q}\bar{Q}' + \alpha z_t z_t' + \beta Q_{t-1} \tag{4}$$

where ω is an $n \times 1$ vector, β and α are $n \times n$ diagonal matrices, Q_t^* is a diagonal matrix containing the diagonal elements of Q_t, $\bar{Q}$ is an $n \times n$ lower triangular matrix, and α and β are scalars. DCC-GARCH can also be estimated using composite likelihood, which is discussed in Section 3.

2.2.2 BEKK-GARCH Model

The other widely used GARCH model that relies only on daily low-frequency data is the BEKK model introduced by Engle and Kroner (1995). Unlike the DCC model, the BEKK model directly models the conditional covariance matrix as follows:

$$r_t = H_t^{1/2}\varepsilon_t, \qquad \varepsilon_t \sim N(0, I_n) \tag{5}$$

$$H_t = CC' + BH_{t-1}B' + Ar_{t-1}r_{t-1}'A' \tag{6}$$

where C is an $n \times n$ lower triangular matrix, and B and A are scalars. C, B, and A are estimated using the composite likelihood method (see Section 3).

2.3 Modeling Using Realized Volatility

2.3.1 MRG Model

Archakov et al. (2020) introduced a multivariate GARCH model that models the conditional covariance matrix using realized variance and correlation. This MRG model is a generalization of the realized beta GARCH model proposed by Hansen et al. (2014) and follows the DCC-GARCH framework that separately models the conditional variance and correlation matrix.

We use the following realized GARCH model to measure the dynamics of the vector of conditional variance and realized variance:

$$r_t = \Lambda_{h_t}^{1/2} z_t, \qquad z_t \sim N(0, C_t) \tag{7}$$

$$\log(h_t) = \omega + \beta\log(h_{t-1}) + \tau(z_{t-1}) + \gamma\log(x_{t-1}) \tag{8}$$

$$\log(x_t) = \xi + \varphi\log(h_t) + \delta(z_t) + v_t \tag{9}$$

Equation (8) models the dynamic property of the conditional variance vector, where ω is an $n \times 1$ vector, β and γ are $n \times n$ matrices, $\tau(\cdot)$ is a leverage function with $\tau(z_t) = \tau_1 z_t + \tau_2(z_t \circ z_t - l_n)$, τ_1 and τ_2 are $n \times n$ matrices, $\circ$ is Hadamard product. Equation (9) is the corresponding measurement equation, where ξ is an $n \times 1$ vector, φ is an $n \times n$ matrix, $\delta(\cdot)$ is a leverage function with $\delta(z_t) = \delta_1 z_t + \delta_2(z_t \circ z_t - l_n)$, δ_1 and δ_2 are $n \times n$ matrices, v_t is a normally distributed error term. We assume that β, γ, τ_1, τ_2, φ, δ_1, and δ_2 are diagonal matrices; for example, $\beta = diag(\beta_{11}, \beta_{22}, \dots\dots, \beta_{nn})$.

To model the conditional correlation and realized correlation, Archakov et al. (2020) used the parameterization approach proposed by Archakov and Hansen (2020) and modeled the vector transformation of the conditional correlation and realized correlation matrix instead of directly modeling the conditional correlation or covariance matrix.

$$\varrho_t = g(C_t) = vecl(\log(C_t))$$

$$y_t = g(Y_t) = vecl(\log(Y_t))$$

This method ensures the positive definiteness of the covariance matrix without imposing additional restrictions. The dynamics are as follows:

$$\varrho_t = \tilde{\omega} + \tilde{\beta}\varrho_{t-1} + \tilde{\gamma}y_{t-1} \tag{10}$$

$$y_t = \tilde{\xi} + \tilde{\varphi}\varrho_t + \tilde{v}_t \tag{11}$$

Equation (10) describes the modeling of the vector representation of the conditional correlation matrix, where $\tilde{\omega}$ is an $N \times 1$ vector. Equation (11) models the realized conditional correlation, where $\tilde{\xi}$ is an $N \times 1$ vector and $\tilde{v}_t$ is a normally distributed error term. We denote $u_t = (v_t', \tilde{v}_t')'$ to stack the error terms in the measurement equations and assume that u_t is i.i.d. $N(0, \Sigma)$, which is independent of z_t. We assume that $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\varphi}$ are scalars to make it easier for the model to converge.

We estimate the parameters using quasi-maximum likelihood estimation (QMLE) and use the two-stage estimation and composite likelihood approaches to speed up the estimation process. Section 3 shows further estimation details.

2.3.2 RWG Model

The RWG model of Gorgi et al. (2019) is a new class of MGARCH models using the realized measure of the covariance matrix. Let s_t be a mean-zero and finite variance martingale difference sequence. The model specification is as follows:

$$r_t = H_t^{1/2}\varepsilon_t, \qquad \varepsilon_t \sim N(0, I_n) \tag{12}$$

$$H_t = \Lambda V_t \Lambda' \tag{13}$$

$$RV_t = V_t^{1/2}\eta_t V_t^{1/2}, \qquad \eta_t \sim W_n(I_n/\nu, \nu) \tag{14}$$

where Λ is an $n \times n$ matrix, ν is a scalar such that $\nu > n$[3], ε_t and η_t are, serially and mutually, i.i.d. processes, and V_t is the mean of the Wishart distribution $W_n(I_n/\nu, \nu)$. The dynamic property of V_t is as follows:

$$V_t = vech(f_t) \tag{15}$$

$$f_{t+1} = \omega + Bf_t + As_t \tag{16}$$

where ω is an $N^* \times 1$ vector, s_t is a finite variance martingale difference sequence, and B and A are scalars. These parameters are estimated using composite likelihood. Section 3 shows further estimation details.

2.4 Direct Modeling

2.4.1 HAR Model

The HAR model proposed by Corsi (2009) is used widely in volatility forecasting with realized measures. Chiriac and Voev (2010) extended the univariate HAR model to a multivariate setting. The scalar version of this model is as follows:

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2 p_{t-5|t-1} + \beta_3 p_{t-22|t-1} + \epsilon_t \tag{17}$$

where $p_t = vech(P_t)$ is the $N^* \times 1$ vector derived by stacking the lower triangular components of matrix P_t, and P_t is the Cholesky decomposition of RV_t, such that $P_t P_t' = RV_t$. $p_{t-h|t-1} = 1/h \sum_{i=1}^{h} p_{t-i}$. β_0 is an $N^* \times 1$ intercept vector, while β_1, β_2, and β_3 are all set to be scalar, which can be easily estimated by ordinary least squares. This specification ensures that the predicted realized covariance matrix is positive definite, and the scalar version improves the running speed while making it easier to converge.

2.4.2 REWMA Model

Longerstaey and Spencer (1996) first introduced the Exponentially Weighted Moving Average (EWMA) filter, which is widely used in practice. Fleming et al. (2003) proposed a Realized EWMA (REWMA) filter that uses intra-day high-frequency data. Let $\hbar_t = vech(H_t)$ and $rv_t = vech(RV_t)$. The model specification is as follows:

$$\hbar_t = (1-\alpha)\hbar_{t-1} + \alpha r v_{t-1} \tag{18}$$

where α is a scalar denoted as the decay rate. When α is low, $\hbar_t$ is more persistent. If we assume that the initial value H_0 is positive definite, then the predicted covariance matrix will be positive definite. While Longerstaey and Spencer (1996) provided a standard choice method for α (α is equal to 0.03 for monthly data and 0.06 for daily data), the parameter can still be estimated using standard QMLE. We use composite likelihood (see Pakel et al., 2021) to estimate α based on the assumption that the daily return vector is conditionally normally distributed to facilitate implementation. Section 3 provides further estimation details.

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[3] To ensure that RV_t is a Wishart distribution.

3. Estimation and Forecasting

3.1 Estimation

We estimate all models using QMLE except the HAR model. For the sake of brevity, the following likelihoods omit the constant terms.

3.1.1 DCC-GARCH Model

We also use the two-stage estimation method. In the first stage, we estimate the conditional variance series using the univariate GARCH model (Equation 2) and get the estimated $\theta_1 = (\omega, diag(\alpha), diag(\beta))$. In the second stage, we take the estimated conditional variance series as fixed and focus on estimating the correlation matrix using the following likelihood function:

$$\log L(\theta_2) = \frac{1}{T}\sum_{t=1}^{T} -\frac{1}{2}(log|C_t| + z_t' C_t^{-1} z_t) \tag{19}$$

where $\theta_2 = (\alpha, \beta)$. We use $(1-\alpha-\beta)(T^{-1}\sum_{t=1}^{T} r_t r_t' \circ diag(h_t)^{-1})$ to estimate $\bar{Q}\bar{Q}'$.

3.1.2 BEKK-GARCH Model

The log-likelihood function is as follows:

$$\log L(\theta) = \frac{1}{T}\sum_{t=1}^{T} -\frac{1}{2}(\log|H_t| + r_t' H_t^{-1} r_t) \tag{20}$$

where $\theta = (A, B)$. We use $(1 - A - B)EH$[4] to estimate CC'.

3.1.3 MRG Model

We apply the two-stage estimation method to get the estimated parameters using the following steps.

1. Use the univariate realized GARCH model (Hansen et al., 2012) to estimate the conditional variance series. The unknown parameters are $\theta_1 = (\theta_{1,1}, \theta_{1,2}, \dots, \theta_{1,n})$, where $\theta_{i,n} = (\omega_i, \beta_{ii}, \gamma_{ii}, \tau_{1_{ii}}, \tau_{2_{ii}}, \xi_i, \varphi_{ii}, \delta_{1_{ii}}, \delta_{2_{ii}}, \sigma_i)$ and $\sigma_i = \Sigma_{ii}$. The likelihood function for $\theta_{i,n}$ is as follows:

$$\log L(\theta_{i,n}) = \frac{1}{T}\sum_{t=1}^{T} -\frac{1}{2}(logh_{i,t} + \frac{u_{i,t}}{\sigma_i^2}) \tag{21}$$

   We use the sample average to fix the values of the initial conditional variance vector h_0. These likelihood functions are directly estimated. We obtain the estimator $\widehat{\theta_1}$ and calculate the time series $\{h_t, z_t, u_t\}, t = 1, \dots, T$.
2. Apply the covariance targeting approach (Engle and Mezrich, 1996) to estimate the intercept parameters using the following empirical moments:

$$\tilde{\omega} = E\varrho - \tilde{\beta}E\varrho - \tilde{\gamma}Ey$$
$$\tilde{\xi} = Ey - \tilde{\varphi}E\varrho$$

   where $E\varrho$ and Ey are the sample averages of the corresponding series $\{\varrho_t, y_t\}$[5]. With this method, we only have to estimate $\theta_2 = (\tilde{\beta}, \tilde{\gamma}, \tilde{\varphi})$. Let $\hat{u}_t$ be the estimation residuals of Equations (10) and (11). It can be shown that the MLE of Σ is $\hat{\Sigma} = T^{-1}\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$, so we can simplify the likelihood function as follows:

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[4] Here, EH is the simple average of $\{r_t r_t'\}$.

[5] Here, we define $\varrho_t = g(r_t r_t' \circ diag(h_t)^{-1})$.

$$\log L(\theta_2) = \frac{1}{T}\sum\nolimits_{t=1}^{T} l_{r,t} + \frac{1}{T}\sum\nolimits_{t=1}^{T} l_{x,y|r,t} \tag{22}$$

$$l_{r,t}(\theta_2) = -\frac{1}{2}(\sum\nolimits_{k=1}^{n} logh_{k,t} + log|C_t| + z_t' C_t^{-1} z_t) \tag{23}$$

$$l_{x,y|r,t}(\theta_2) = -\frac{1}{2} log\left|\frac{1}{T}\sum\nolimits_{t=1}^{T} u_t u_t'\right| \tag{24}$$

To satisfy the stationarity condition, we require that $\max(\text{diag}(\beta + \gamma\varphi)) < 1$ and $\tilde{\beta} + \tilde{\gamma}\tilde{\varphi} < 1$. We do not have to impose any restrictions to ensure the positive definiteness of the correlation matrix (Archakov and Hansen, 2020).

3.1.4 RWG Model

Gorgi et al. (2019) used the score-driven method to describe the dynamics of s_t; we can derive the log-likelihood function as follows:

$$\log L(\theta) = \frac{1}{T}\sum\nolimits_{t=1}^{T} l_{r,t} + \frac{1}{T}\sum\nolimits_{t=1}^{T} l_{RV,t} \tag{25}$$

$$l_{r,t}(\theta) = -\frac{1}{2}(log|\Lambda V_t \Lambda'| + tr(\Lambda V_t \Lambda')^{-1} r_t r_t') \tag{26}$$

$$l_{RV,t}(\theta) = \frac{\nu - n - 1}{2} log|RV_t| - \frac{\nu}{2} log|V_t| - \frac{\nu}{2} tr(V_t^{-1} RV_t) \tag{27}$$

where $\theta = (\nu, A, B, diag(\Lambda))$. We use $(1 - A - B)vech(T^{-1}\sum_{t=1}^{T} RV_t)$ to fix the value of ω. All of the parameters are estimated using the composite likelihood method discussed in Section 3.2.

3.1.5 REWMA Model

In Section 2.4.2, we assume that the return vector r_t is conditionally normally distributed with a mean of 0 and a covariance matrix of H_t. The only parameter is α, so the likelihood function for REWMA is:

$$\log L(\alpha) = \frac{1}{T}\sum\nolimits_{t=1}^{T} -\frac{1}{2}(\log|H_t| + r_t' H_t^{-1} r_t) \tag{28}$$

The log-likelihood is estimated using the composite likelihood approach (see Section 3.2).

3.2 Composite Likelihood Approach

The composite likelihood approach of Pakel et al. (2021) is widely used in estimating large dimensional volatility models. The basic idea of the approach is as follows. Let H_t be the $n \times n$ conditional covariance matrix of the $n \times 1$ return vector, r_t. Assuming that r_t is conditionally normally distributed, the standard quasi-likelihood function is

$$\log(L(\theta; r)) = \frac{1}{T}\sum_{t=1}^{T} -\frac{1}{2}(\log(|H_t|) + r_t' H_t^{-1} r_t) \tag{29}$$

We have to calculate the determinant and inverse of the $n \times n$ conditional covariance matrix. When n is large, the estimation process is computationally costly and the estimators are meaningfully biased (Engle, 2009). However, the composite likelihood approach prevents these issues. Instead of directly estimating the whole likelihood function in Equation (29), the composite likelihood approach approximates the likelihood function with a number of two-dimensional marginal densities, thus changing an n-dimensional problem into a few two-dimensional problems.

Here, we use the all contiguous-pair likelihoods to approximate Equation (29). Let $R_{1,t} = (r_{1,t}, r_{2,t})$, $R_{2,t} = (r_{2,t}, r_{3,t})$, …, $R_{n-1,t} = (r_{n-1,t}, r_{n,t})$, and $H_{j,t}$ be the conditional covariance matrix of the return pair $R_{j,t}$, so the composite likelihood is

$$CL(\theta; r) = \frac{1}{T(n-1)} \sum_{j=1}^{n-1} \sum_{t=1}^{T} -\frac{1}{2} (\log(|H_{j,t}|) + R'_{j,t} H_{j,t}^{-1} R_{j,t}) \tag{30}$$

By solving this composite likelihood, we obtain consistent and asymptotically normal estimators.

3.3 Forecasting

We use a rolling window to estimate the parameters and rebalance forecasting and portfolio selection at the short (daily) and long (weekly and monthly) horizons, which means that the model parameters are updated daily, weekly, and monthly.

Our estimation window consists of 1,991 observations. The out-of-sample period is from January 1, 2009 to 1 December 31, 2009. The daily horizon means that for each trading day in the out-of-sample period, we estimate all of the model parameters using the last 1,991 known observations. After parameter estimation, we calculate the one-step-ahead predicted covariance matrices. The weekly and monthly horizons indicate that we estimate the parameters and rebalance portfolio selection every week or month and obtain forecasts over the weekly and monthly horizons.

3.3.1 Forecasting Procedure

One-step ahead forecasts are quite straightforward because H_{t+1} is $\mathcal{F}_t$-measurable. After observing $\mathcal{F}_t$ and estimating the parameters for period *t*+1, H_{t+1} can be directly computed from the dynamic equations.

We define the forecasts over the weekly and monthly horizons as follows:

$$\widehat{H}_{t+k|t+1} = \frac{1}{k} \sum_{s=1}^{k} \widehat{H}_{t+s} \tag{31}$$

where $\widehat{H}_{t+s}$ denotes *s*-step ahead covariance matrix forecasts and $k = 5$ or 22.

In REWMA, we directly calculate multi-step forecasts because the model does not have an error term. We assume that the conditional expectation of the realized covariance matrix is equal to that of the conditional covariance matrix, that is, $E(\hbar_{t+s}|\mathcal{F}_t) = E(rv_{t+s}|\mathcal{F}_t)$, for $s = 1,2,\ldots,k$. Thus, the *s*-step ahead forecasts are $\hat{h}_{t+s} = E(\hbar_{t+s}|\mathcal{F}_t) = E(\hbar_{t+s-1}|\mathcal{F}_t) = \cdots = \hbar_{t+1}$. As $\hbar_{t+1}$ is $\mathcal{F}_t$-measurable, the forecasts over the weekly and monthly horizons are $\widehat{H}_{t+k|t+1} = H_{t+1}$.

However, the multi-step forecasts in the other models depend on the future realization of some error item series, such as z_t and u_t, which is not straightforward. Hence, we use the bootstrap method of Lunde and Olesen (2014) as the multi-step forecasting scheme. We draw M = 1,000 re-samples of the error term series from the estimated models and iteratively calculate the multi-step forecasts as follows, taking the MRG model as an example:

1. Determine the *s*-step ahead forecasts of this model depending on the error term series $\{(\hat{z}_{t+1}, \hat{u}_{t+1}), \ldots, (\hat{z}_{t+s-1}, \hat{u}_{t+s-1})\}$
2. Generate the required error term series $\{(\hat{z}_{t+1}^m, \hat{u}_{t+1}^m), \ldots, (\hat{z}_{t+s-1}^m, \hat{u}_{t+s-1}^m)\}, m = 1,2,\ldots,M$ from the bootstrap estimation results $\{(\hat{z}_1, \hat{u}_1), \ldots, (\hat{z}_t, \hat{u}_t)\}$
3. Use each generated error term series to iteratively calculate the *s*-step ahead forecasts and calculate the average of these *s*-step ahead forecasts as, $\widehat{H}_{t+s} = \frac{1}{M} \sum_{m=1}^{M} \widehat{H}_{t+s}^m$
4. Obtain forecasts over the weekly and month horizons based on Equation (31)

4. Data

We use intra-day high-frequency and daily low-frequency data on 10 Dow Jones stocks: Bank of America (BAC), JP Morgan Chase (JPM), Alcoa (AA), American Express (AXP), Microsoft (MSFT), Exxon Mobil (XOM), Dupont de Nemours (DD), General Electric (GE), and Coca-Cola (KO). The names are shown in Table 1. We extract 5-minute returns from the New York Stock Exchange's Daily Trade and Quote database. The data span from February 1, 2001 to December 31, 2009, with a total of 2,242 daily observations.

We exclude the opening and closing 15 minutes of a trading day to prevent the overnight effects and then use 5-minute returns with subsampling to calculate the realized covariance matrix. We focus our analysis on open-to-close returns to ensure consistency in the interval of daily returns and the realized covariance matrix. The summary statistics of returns and the realized covariance matrix is shown in Tables 1 and 2. Note that we use demeaned returns in our models.

Table 1. Summary Statistics for Daily Returns and Realized Variance

	BAC	JPM	IBM	MSFT	XOM	AA	AXP	DD	GE	KO
Panel A: Daily returns (× 100)										
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD	2.39	2.20	1.30	1.44	1.29	2.11	2.08	1.45	1.64	1.06
Skew	0.33	0.58	0.01	0.25	-0.19	-0.68	0.32	0.03	0.22	0.10
Kurt	21.81	17.05	6.32	6.16	11.64	9.90	11.23	7.28	10.96	6.92
Min	-19.36	-17.02	-6.60	-8.20	-12.56	-17.08	-15.69	-9.48	-10.67	-7.39
Max	19.69	22.11	7.66	8.78	9.70	12.01	14.79	7.85	12.62	6.40
Panel B: Realized variance (in annual units)										
Mean	0.25	0.28	0.19	0.22	0.20	0.30	0.26	0.22	0.23	0.17
SD	0.27	0.22	0.11	0.12	0.11	0.18	0.20	0.12	0.17	0.09
Skew	3.44	2.70	2.76	1.96	3.86	2.98	2.39	2.41	2.97	2.83
Kurt	18.19	14.59	15.46	8.91	31.02	17.01	12.88	12.97	16.22	18.14
Min	0.04	0.05	0.04	0.05	0.06	0.09	0.04	0.06	0.05	0.03
Max	2.64	2.11	1.20	1.04	1.71	2.01	2.26	1.27	1.70	1.19

Note: Realized variance in annual units is defined as $\sqrt{252} \times \sqrt{x_t}$. The statistics in this table are based on the sample period from February 1, 2002 to December 31, 2009.

Tables 1 and 2 show the summary statistics of open-to-close returns and the realized covariance matrix. Returns in 2008 are more volatile because of the global financial crisis, thereby eliminating the previously accumulated asset returns. According to the standard deviation of returns and the mean realized variance, KO has the lowest volatility while BAC, JPM, AA, and AXP have the highest volatility. As for the average realized correlation, BAC and JPM have the strongest correlation because both are financial firms.

Table 2. Average Realized Correlations

	BAC	JPM	IBM	MSFT	XOM	AA	AXP	DD	GE	KO
BAC										
JPM	0.50									
IBM	0.38	0.38								
MSFT	0.37	0.38	0.44							
XOM	0.33	0.33	0.35	0.35						
AA	0.30	0.31	0.31	0.30	0.34					
AXP	0.44	0.44	0.38	0.37	0.33	0.31				
DD	0.37	0.37	0.38	0.36	0.36	0.37	0.37			
GE	0.41	0.40	0.41	0.40	0.37	0.32	0.40	0.39		
KO	0.31	0.31	0.33	0.32	0.30	0.25	0.31	0.32	0.34	

Note: Realized correlations are defined in Section 2.1. The statistics in this table are based on the sample period from February 1, 2002 to December 31, 2009.

5. Empirical Analysis

In this section, we conduct our empirical analysis to explore the necessity of using high-frequency data under different forecast horizons at the statistical and economic levels. Section 5.1 presents the statistical evaluation method for comparing the different models, especially their ability to forecast covariance. Section 5.2 elaborates on economic loss functions based on portfolio selection.

5.1 Statistical Evaluation

This part discusses how to evaluate the accuracy of conditional covariance matrix forecasts based on a range of robust statistical loss functions. We need to specify the measure of the true conditional covariance matrices to evaluate the differences in the forecasts of our models. In this study, we use the realized covariance as the true conditional covariance matrix.

5.1.1 Statistical Loss Functions

Following Patton and Sheppard (2009) and Laurent et al. (2013), we use robust[6] loss functions, such as the MSE and QLIKE robust loss functions, as follows:

$$\mathcal{L}^{MSE} = \frac{1}{N^2} vec\left(\widehat{H}_t - RV_t\right)' vec(\widehat{H}_t - RV_t) \qquad (32)$$

$$\mathcal{L}^{QLIKE} = log|\widehat{H}_t| + tr(\widehat{H}_t^{-1} RV_t) \qquad (33)$$

where $\widehat{H}_t$ denotes the predicted conditional covariance matrix, RV_t is the realized covariance matrix and the measure of the true conditional covariance matrix, $vec(\cdot)$ represents the column stacking operator, and $tr(\cdot)$ denotes the trace of the matrix. Small loss function values indicate high forecast accuracy. MSE is a symmetric loss function that gives the same penalty for volatility overestimation and underestimation, while QLIKE is an asymmetric loss function that imposes a larger penalty for volatility underestimation. Thus, the evaluation results of QLIKE are more reliable for risk management and portfolio selection.

[6] According to Pattern (2009), "robust" means that the results for volatility comparison are the same irrespective of using true volatility or some conditional unbiased volatility proxy.

We apply the MCS approach of Hansen et al. (2011) to compare the model forecasts to investigate whether the values of these two loss functions significantly differ across our models. This method provides a set of models, which contain the best forecasting model with a given probability (we set it to 90%). The result of the MCS approach is a set of p-values, one for each model. If a model's p-value, denoted by $p_{MCS}(\mathcal{L}^x)$, is below 0.1, the corresponding model is excluded from the best model subset. This subset of models may contain all of the models or none.

5.1.2 Out-of-sample Forecast Evaluation

The out-of-sample forecast evaluation is based on short (daily) and long (weekly and monthly) forecast horizons. The daily forecast horizon requires the parameters to be updated on each trading day of the out-of-sample period using the last 1,991 known observations and forecasting the corresponding 1-day ahead covariance matrix. Under the weekly and monthly forecast horizons, we estimate the models and forecast covariance matrices over the specific horizon.

Tables 3, 4, and 5 present the results of the daily, weekly, and monthly forecast horizons, respectively, for our statistical evaluation of forecast accuracy. The models are divided into three groups. The second and third groups of models both rely on intra-day realized volatility and daily information to forecast the daily covariance matrix. The difference between these two types of models is that "Direct Modeling" only focuses on the covariance matrix forecast but "Modeling with Realized Volatility" also characterizes the daily return distribution.

Table 3 shows that the forecasts of models using high-frequency information are more accurate than those of models using only low-frequency daily information. When we use MSE as the loss function, only the BEKK model is excluded from the 90% MCS. Although the DCC model is also included in the 90% MCS, its MCS p-value is the smallest out of all of the models included, and the mean value of the loss function is also the largest among all included models, meaning that the forecast error of the DCC model is higher than that of the other models incorporating intra-day realized volatility. When we use QLIKE as the loss function, only the REWMA model is included in the 90% MCS, while the BEKK and DCC models are the first two models to be excluded from the MCS in the corresponding algorithm. Thus, the 1-day ahead forecast performance of models using only daily information (DCC and BEKK) is worse than that of models using intra-day high-frequency information. In other words, using high-frequency data significantly improves forecast accuracy over the daily horizon.

Table 3. Out-of-sample Forecasts over the Daily Horizon, Comparison Results

	Modeling with Daily Information		Modeling with Realized Volatility		Direct Modeling	
	DCC	BEKK	MRG	RWG	HAR	REWMA
$\overline{\mathcal{L}^{MSE}}$	**21.152**	24.430	**16.441**	**17.344**	**16.978**	**16.806**
$p_{MCS}(\mathcal{L}^{MSE})$	0.106	0.033	1.000	0.482	0.577	0.577
$\overline{\mathcal{L}^{QLIKE}}$	18.924	20.118	18.957	18.674	18.393	**18.252**
$p_{MCS}(\mathcal{L}^{QLIKE})$	0.000	0.000	0.000	0.000	0.007	1.000

Note: The table reports the statistical evaluation of short-horizon (daily) forecasts. $\overline{\mathcal{L}^{MSE}}$ and $\overline{\mathcal{L}^{QLIKE}}$ represent the average values of the MSE and QLIKE loss functions. $p_{MCS}(\mathcal{L}^{MSE})$ and $p_{MCS}(\mathcal{L}^{QLIKE})$ denote the MCS p-values based on the values of the two robust loss functions. When the MCS p-value is small, the model is more likely to be excluded from the 90% MCS. All entries in **boldface** indicate that the models are included in the 90% MCS for that row.

Although the forecasts of models using realized volatility are more accurate over the daily horizon, this improvement become nonsignificant over longer horizons. Tables 4 and 5 show the forecast results for the weekly and monthly horizons. The results for the long forecast horizons average all possible weekly (Monday-to-Monday, Tuesday-to-Tuesday, etc.) and monthly (1st day-to-1st day, 2nd day-to-2nd day, etc.) combinations. Tables 4 and 5 show that the benefit of using high-frequency data diminishes with a longer forecast horizon. The average value of the MSE loss function for models using only daily information is higher than that of models using realized volatility. The DCC and BEKK models are both excluded from the 90% MCS over the weekly and monthly forecast horizons. Hence, reducing the forecast frequency to weekly and monthly rebalancing does not change the conclusion we draw from Table 3. However, when using the QLIKE loss function, we find that the difference in forecast accuracy between models using daily and intra-day information becomes smaller. The forecast error of the BEKK model is less than that of the RWG model. The average loss function value of the DCC model is lower than that of the MRG, RWG, and HAR models. Under the monthly forecast horizon, only the DCC and REWMA models are included in the 90% MCS. As the QLIKE loss function is more suitable for analyzing risk management-related topics, our main results are based on the QLIKE loss function. Thus, a long forecast horizon reduces the improvements in forecast accuracy resulting from the use of high-frequency data.

Table 4. Out-of-sample Forecasts over the Weekly Horizon, Comparison Results

	Modeling with Daily Information		Modeling with Realized Volatility		Direct Modeling	
	DCC	BEKK	MRG	RWG	HAR	REWMA
$\overline{\mathcal{L}^{MSE}}$	14.119	15.411	**10.070**	**10.620**	**10.397**	**10.215**
$p_{MCS}(\mathcal{L}^{MSE})$	0.009	0.003	1	0.360	0.707	0.732
$\overline{\mathcal{L}^{QLIKE}}$	18.375	18.652	18.640	18.749	18.396	**18.247**
$p_{MCS}(\mathcal{L}^{QLIKE})$	0.000	0.000	0.000	0.000	0.002	1.000

Note: The table reports the statistical evaluation of long-horizon (weekly) forecasts. $\overline{\mathcal{L}^{MSE}}$ and $\overline{\mathcal{L}^{QLIKE}}$ represent the average values of the MSE and QLIKE loss functions. $p_{MCS}(\mathcal{L}^{MSE})$ and $p_{MCS}(\mathcal{L}^{QLIKE})$ denote the MCS p-values based on the values of the two robust loss functions. When the MCS p-value is small, the model is more likely to be excluded from the 90% MCS. All entries in **boldface** indicate that the models are included in the 90% MCS for that row.

Table 5. Out-of-sample Forecasts over the Monthly Horizon, Comparison Results

	Modeling with Daily Information		Modeling with Realized Volatility		Direct Modeling	
	DCC	BEKK	MRG	RWG	HAR	REWMA
$\overline{\mathcal{L}^{MSE}}$	14.102	13.527	10.646	**9.563**	10.750	10.764
$p_{MCS}(\mathcal{L}^{MSE})$	0	0	0.006	1	0.006	0.033
$\overline{\mathcal{L}^{QLIKE}}$	**18.375**	18.652	18.640	18.749	18.396	**18.247**
$p_{MCS}(\mathcal{L}^{QLIKE})$	0.134	0.000	0.000	0.000	0.061	1.000

Note: The table reports the statistical evaluation of short-horizon (monthly) forecasts. $\overline{\mathcal{L}^{MSE}}$ and $\overline{\mathcal{L}^{QLIKE}}$ represent the average values of the MSE and QLIKE loss functions. $p_{MCS}(\mathcal{L}^{MSE})$ and $p_{MCS}(\mathcal{L}^{QLIKE})$ denote the MCS p-values based on the values of the two robust loss functions. When the MCS p-value is small, the model is more likely to be excluded from the 90% MCS. All entries in **boldface** indicate that the models are included in the 90% MCS for that row.

5.2 Economic Evaluation

The economic evaluation of the three types of models focuses on the use of forecasts in the construction of an optimal portfolio. From an economic perspective, the portfolios thus obtained should have excellent properties.

The optimal portfolio is defined as the global minimum variance (GMV) portfolio. GMV portfolios are typically used in the economic evaluation of volatility models (Bollerslev et al., 2018; Ledoit and Wolf, 2018; Engle et al., 2019). Estimating the GMV portfolio is a clean framework for evaluating covariance forecasts because it prevents us from estimating the expected return vector, whose estimation error distorts the optimal weights. Furthermore, the out-of-sample properties (i.e., in terms of the Sharpe ratio) of GMV portfolios usually outperform mean-variance portfolios (Jagannathan and Ma, 2003; DeMiguel et al., 2009).

5.2.1 GMV Portfolio

Investors follow a dynamic strategy over a fixed investment horizon, in which the model parameters are updated, the conditional covariance matrices, $\widehat{H}_{t+h|t+1}$, are predicted over the investment horizon, and the optimal portfolio weights, w_t, are adjusted correspondingly. We assume that investors do not face transaction costs and short-selling constraints as follows:

$$\min_{w_t} \; w_t' \widehat{H}_{t+h|t+1} w_t \tag{34}$$

$$s.t. \, w_t' l = 1 \tag{35}$$

where h denotes the daily, weekly, or monthly investment horizon, $\widehat{H}_{t+h|t+1}$ is the predicted covariance matrix over the corresponding horizon, w_t is the portfolio weight determined on trading day t and unchanged from trading day t to trading day $t+h-1$, and l is the $n \times 1$ vector of ones. Equation (35) ensures that the sum of the portfolio weights is 1. From Equations (34) and (35), we obtain the explicit solution of the optimal portfolio weight as follows:

$$w_t = \frac{\widehat{H}_{t+h|t+1}^{-1} l}{l' \widehat{H}_{t+h|t+1}^{-1} l} \tag{36}$$

Next, we denote the ith element of w_t and r_t by $w_{i,t}$ and $r_{i,t}$, respectively, and the weight based on the mth model's forecasts by $w_t^{(m)}$.

5.2.2 Economic Loss Functions

We discuss the economic loss functions used in this study to evaluate the performance of the selected portfolios based on the covariance forecasts of the different models. Next, we introduce six economic loss functions: portfolio variance, MSE weight, utility-based framework, turnover, concentration, and short positions.

We directly compare the portfolio variances during the out-of-sample period to evaluate forecasting performance (Chiriac and Voev, 2011). For each horizon in the out-of-sample period, the optimal portfolio weight is derived using different forecasts. We use the realized covariance as the true conditional covariance matrix of the return vector r_t and calculate the variance of the portfolio as follows:

$$\mathcal{L}_1 = \sigma_{p,t+h|t+1}^2 = w_t' RV_{t+h|t+1} w_t$$

Another related criterion used for volatility evaluation is the MSE weight (Golosnoy and Gribisch, 2022):

$$\mathcal{L}_2 = (w_t - w_t^*)'(w_t - w_t^*)$$

where w_t^* denotes the ex-post realization—the weight calculated after knowing the true conditional covariance matrix.

The third economic loss function we use is the utility-based framework of Fleming et al. (2003). They assumed that investors have quadratic utility, whereas we use the negative value of this utility as our third economic loss function as follows:

$$\mathcal{L}_3 = -U(r_{p,t};\gamma) = -\left[\ \left(1 + r_{p,t}\right) - \frac{\gamma}{2(1+\gamma)}\left(1 + r_{p,t}\right)^2\ \right]$$

where $r_{p,t}$ is the portfolio return on trading day t, and γ is the risk aversion set to 1.

We also consider other widely used evaluation frameworks. According to DeMiguel et al. (2009), Callot et al. (2017), Bollerslev et al. (2018), and Grønborg et al. (2022), total portfolio turnover measures the change in trading volume. Low turnover indicates low transaction costs (proportional transaction costs). We define turnover as follows:

$$\mathcal{L}_4 = TO_t = \sum_{i=1}^{n} \left| w_{i,t+1} - w_{i,t} \frac{1 + r_{i,i}}{1 + w_t' r_t} \right|$$

The next economic loss function we use is portfolio concentration, which measures extreme portfolio allocation risk. Following Bollerslev et al. (2018), portfolio concentration is calculated as follows:

$$\mathcal{L}_5 = CO_t = \left(\sum_{i=1}^{n} w_{i,t}^2\right)^{1/2}$$

Bollerslev et al. (2018) and Grønborg et al. (2022) also used total portfolio short positions because the implementation of short positions is costlier than that of long positions.

$$\mathcal{L}_6 = SP_t = \sum_{i=1}^{n} w_{i,t}\mathbb{I}(w_{i,t} < 0)$$

where $\mathbb{I}(\cdot)$ is the indicator function.

5.2.3 Economic Performance Results

Similar to the statistical evaluation in Section 5.2, our economic evaluation is based on daily, weekly, and monthly horizons. We evaluate the performance of the covariance forecasts based on the above economic loss functions. Tables 6, 7, and 8 show the daily, weekly, and monthly results, respectively.

Realized volatility cannot significantly reduce all of the economic loss functions in the daily investment horizon; that is, the economic improvement in high-frequency data depends on the evaluation framework. High-frequency data can make the optimal weight more accurate, thus effectively reducing out-of-sample portfolio variances. However, its performance of high-frequency data under the utility and portfolio structure (TO, CO, and SP) frameworks is not significantly better than that using low-frequency data. From Table 6, the MCS p-values of direct modeling are higher than those of the other models, indicating that the HAR and REWMA models significantly reduce out-of-sample portfolio variance. The portfolios generated by models using daily information have the largest out-of-sample portfolio variance. Moreover, modeling with realized volatility constructs portfolios closest to the ex-post optimal realization and is included in the 90% MCS, while the weight of models using daily information has the highest MSE of all ex-post optimal weights. Thus, the models using high-frequency data significantly outperform those using daily information in terms of portfolio variance and MSE weights in the daily investment horizon. However, when focusing on other economic losses, all of the models are included in the 90% MCS of negative utility, and the DCC model has the highest MCS p-value. Furthermore, the 90% MCS of turnover loss only has the DCC model, which indicates that the DCC model significantly reduces transaction costs in reality compared with the other models, even over the daily investment horizon. In addition, the DCC, RWG, and REWMA models are all included in the 90% MCS of portfolio concentration and short-selling position losses, which means that high-frequency data cannot significantly reduce these losses related to portfolio structures. Thus, high-frequency data improve portfolio performance in terms of variance and MSE weights but not in terms of investor utility and portfolio structures.

Table 6. Out-of-sample Economic Evaluation over the Daily Horizon

	Modeling with Daily Information		Modeling with Realized Volatility		Direct Modeling	
	DCC	BEKK	MRG	RWG	HAR	REWMA
$\overline{\sigma_p}$	0.898	1.130	0.847	0.822	0.820	**0.814**
$p_{MCS}(\sigma_p)$	0.005	0.005	0.011	0.011	0.069	1
$\overline{w^{MSE}}$	0.070	0.073	0.036	**0.032**	0.041	0.052
$p_{MCS}(w^{MSE})$	0.000	0.000	0.000	0.000	0.000	0.000
$-\overline{U(r_p,\gamma)}$	**-0.572**	**-0.550**	**-0.561**	**-0.566**	**-0.565**	**-0.572**
$p_{MCS}(U(r_p,\gamma))$	1.000	0.671	0.671	0.671	0.671	0.987
$\overline{TO}$	**0.152**	0.167	0.270	0.292	0.360	0.231
$p_{MCS}(TO)$	1.000	0.019	0.000	0.000	0.000	0.000
$\overline{CO}$	**0.672**	0.729	0.688	**0.661**	0.676	**0.661**
$p_{MCS}(CO)$	0.365	0.000	0.000	0.667	0.000	1.000
$\overline{SP}$	**0.210**	0.329	0.228	**0.197**	0.210	**0.197**
$p_{MCS}(SP)$	0.574	0.000	0.000	0.878	0.000	1.000

Note: The table reports the economic evaluation of short-horizon (daily) forecasts. $\overline{\sigma_p}$, $\overline{w^{MSE}}$, $-\overline{U(r_p,\gamma)}$, $\overline{TO}$, $\overline{CO}$, and $\overline{SP}$ respectively represent the average values of out-of-sample portfolio variance, MSE weight, negative utility, turnover, concentration, and short position. $p_{MCS}(\cdot)$ denotes the corresponding MCS p-values of these economic loss functions. When the MCS p-value is small, the model is more likely to be excluded from the 90% MCS. All entries in **boldface** indicate that the models are included in the 90% MCS for that row.

Similarly, in the weekly and monthly horizons, investors cannot obtain significant economic gains from realized volatility. High-frequency data can only reduce portfolio variance but are mediocre in other aspects compared with the models using daily information over the weekly and monthly investment horizons. The results for the long forecast horizons average all possible weekly (Monday-to-Monday, Tuesday-to-Tuesday, etc.) and monthly (1^{st} day-to-1^{st} day, 2^{nd} day-to-2^{nd} day, etc.) combinations. Tables 7 and 8 show that the HAR/REWMA and MRG/HAR/REWMA models perform best in terms of portfolio variance over the weekly and monthly investment horizons. As for the MSE weights, the RWG/HAR models are included in the 90% MCS. These results related to portfolio variance and MSE weights are consistent with those over the daily investment horizon. Nonetheless, the role of high-frequency data in portfolio performance changes when we concentrate on other economic losses. No models are excluded from the 90% MCS of investor utility. The BEKK model is the best in terms of turnover with weekly rebalancing frequency. Moreover, the DCC model is one of the best models regarding portfolio concentration and selling position over the weekly and monthly investment horizons. Thus, a long investment horizon does not change the results of the short investment horizon—that high-frequency data significantly enhance portfolio performance with regard to portfolio variance, while the role of high-frequency data in promoting investor utility and refining portfolio structures is no different from that of low-frequency data.

Table 7. Out-of-sample Economic Evaluation over the Weekly Horizon

	Modeling with Daily Information		Modeling with Realized Volatility		Direct Modeling	
	DCC	BEKK	MRG	RWG	HAR	REWMA
$\overline{\sigma_p}$	0.896	1.083	0.841	0.833	**0.821**	**0.821**
$p_{MCS}(\sigma_p)$	0.001	0.001	0.010	0.010	0.810	1.000
$\overline{w^{MSE}}$	0.018	0.022	0.010	**0.009**	**0.010**	0.011
$p_{MCS}(w^{MSE})$	0.000	0.000	0.022	1.000	0.274	0.016
$\overline{U(r_p,\gamma)}$	**-0.729**	**-0.731**	**-0.728**	**-0.739**	**-0.736**	**-0.739**
$p_{MCS}(U(r_p,\gamma))$	0.501	0.747	0.501	0.844	0.747	1.000
$\overline{TO}$	0.263	**0.245**	0.257	0.297	0.244	**0.231**
$p_{MCS}(TO)$	0.000	0.229	0.003	0.000	0.003	1.000
$\overline{CO}$	**0.672**	0.718	0.681	**0.660**	0.673	**0.661**
$p_{MCS}(CO)$	0.329	0.000	0.000	1.000	0.000	0.685
$\overline{SP}$	**0.206**	0.309	0.219	**0.195**	0.205	**0.197**
$p_{MCS}(SP)$	0.559	0.000	0.002	1.000	0.046	0.559

Note: The table reports the economic evaluation of long-horizon (weekly) forecasts. $\overline{\sigma_p}$, $\overline{w^{MSE}}$, $-\overline{U(r_p,\gamma)}$, $\overline{TO}$, $\overline{CO}$, and $\overline{SP}$ respectively represent the average values of out-of-sample portfolio variance, MSE weight, negative utility, turnover, concentration, and short position. $p_{MCS}(\cdot)$ denotes the corresponding MCS p-values of these economic loss functions. When the MCS p-value is small, the model is more likely to be excluded from the 90% MCS. All entries in **boldface** indicate that the models are included in the 90% MCS for that row.

Table 8. Out-of-sample Economic Evaluation over the Monthly Horizon

	Modeling with Daily Information		Modeling with Realized Volatility		Direct Modeling	
	DCC	BEKK	MRG	RWG	HAR	REWMA
$\overline{\sigma_p}$	0.820	0.861	**0.788**	0.799	**0.780**	**0.786**
$p_{MCS}(\sigma_p)$	0.011	0.003	0.122	0.058	1.000	0.179
$\overline{w^{MSE}}$	0.013	0.014	0.010	**0.008**	**0.008**	0.010
$p_{MCS}(w^{MSE})$	0.006	0.008	0.017	0.401	0.000	0.087
$\overline{U(r_p,\gamma)}$	**-0.761**	**-0.757**	**-0.759**	**-0.765**	**-0.763**	**-0.767**
$p_{MCS}(U(r_p,\gamma))$	0.453	0.437	0.437	0.453	0.453	1.000
$\overline{TO}$	0.263	0.220	0.215	0.306	**0.180**	0.230
$p_{MCS}(TO)$	0.000	0.000	0.000	0.000	1.000	0.000
$\overline{CO}$	**0.657**	0.675	**0.659**	**0.653**	0.667	0.661
$p_{MCS}(CO)$	0.620	0.000	0.620	1.000	0.001	0.006
$\overline{SP}$	**0.189**	0.235	**0.192**	**0.187**	0.197	0.197
$p_{MCS}(SP)$	0.895	0.000	0.850	1.000	0.017	0.017

Note: The table reports the economic evaluation of long-horizon (monthly) forecasts. $\overline{\sigma_p}$, $\overline{w^{MSE}}$, $-\overline{U(r_p,\gamma)}$, $\overline{TO}$, $\overline{CO}$, and $\overline{SP}$ respectively represent the average values of out-of-sample portfolio variance, MSE weight, negative utility, turnover, concentration, and short position. $p_{MCS}(\cdot)$ denotes the corresponding MCS p-values of these economic loss functions. When the MCS p-value is small, the model is more likely to be excluded from the 90% MCS. All entries in **boldface** indicate that the models are included in the 90% MCS for that row.

6. Conclusion

In this study, we investigate the role of using high-frequency data in covariance forecasts and portfolio performance under different rebalancing frequencies based on three types of models. We use the composite likelihood approach to estimate the required model parameters and forecast the corresponding covariance matrices over daily, weekly, and monthly horizons. We statistically evaluate the forecast accuracy of the different models based on the QLIKE loss function and find that high-frequency data significantly improve forecast accuracy over the daily rebalancing frequency, but this improvement is dampened when longer (weekly and monthly) horizons are used. We also evaluate forecasts from an economic perspective and demonstrate that investors may not obtain significant economic benefits from using high-frequency data, depending on the type of economic loss they pay attention to. In the daily investment horizon, high-frequency data can make the optimal weight more accurate, thus reducing out-of-sample portfolio variance. However, investors cannot enhance their utility or refine their portfolio structure by using high-frequency data. These results are robust to weekly and monthly investment horizons.

Reference

[1] Amendola, A., Braione, M., Candila, V., & Storti, G., "A model confidence set approach to the combination of multivariate volatility forecasts", *International Journal of Forecasting*, 2020, 36(3), 873-891.

[2] Andersen T G, Bollerslev T. "Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts", *International economic review*, 1998: 885-905.

[3] Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys, P., "Modeling and Forecasting Realized Volatility", *Econometrica*, 2003, 71(2), 579-625.

[4] Archakov, I., and Hansen, P. R., "A New Parametrization of Correlation Matrices", *Econometrica*, 2021, 89(4), 1699-1715.

[5] Archakov, I., Hansen, P. R., and Lunde, A., "A Multivariate Realized GARCH Model", 2020, *arXiv preprint arXiv:2012.02708*.

[6] Barndorff‐Nielsen, O. E., and Shephard, N., "Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics", *Econometrica*, 2004, 72(3), 885-925.

[7] Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., and Shephard, N., "Multivariate Realised Kernels: Consistent Positive Semi-Definite Estimators of the Covariation of Equity Prices with Noise and Non-Synchronous Trading", *Journal of Econometrics*, 2011, 162(2), 149-169.

[8] Bollerslev, T., "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 1986, 31(3), 307-327.

[9] Bollerslev, T., "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model", 1990, *Review of Economics and Statistics*, 498-505.

[10] Bollerslev, T., Patton, A. J., and Quaedvlieg, R., "Modeling and Forecasting (un) Reliable Realized Covariances for more Reliable Financial Decisions", *Journal of Econometrics*, 2018, 207(1), 71-91.

[11] Callot, L. A., Kock, A. B., and Medeiros, M. C., "Modeling and Forecasting Large Realized Covariance Matrices and Portfolio Choice", *Journal of Applied Econometrics*, 2017, 32(1), 140-158.

[12] Chiriac, R., and Voev, V., "Modelling and Forecasting Multivariate Realized Volatility", *Journal of Applied Econometrics*, 2010, 26(6), 922-947.

[13] DeMiguel, V., Garlappi, L., and Uppal, R., "Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?", *The Review of Financial Studies*, 2009, 22(5), 1915-1953.

[14] De Nard, G., Engle, R. F., Ledoit, O., & Wolf, M, "Large dynamic covariance matrices: Enhancements based on intraday data", *Journal of Banking & Finance*, 2022, 138, 106426.

[15] Engle, R., "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business and Economic Statistics*, 2002a, 20(3), 339-350.

[16] Engle R., "New Frontiers for ARCH Models", *Journal of Applied Econometrics*, 2002b, 17(5): 425-446.

[17] Engle, R. F., "High Dimension Dynamic Correlations", The Methodology and Practice of Econometrics: Festscrift for David Hendry', Oxford University Press, USA, 2009, 122-148.

[18] Engle R F, and Gallo G M., "A Multiple Indicators Model for Volatility Using Intra-Daily Data",

Journal of Econometrics, 2006, 131(1-2): 3-27.
[19] Engle, R. F., Ghysels, E., & Sohn, B., "Stock Market Volatility and Macroeconomic Fundamentals", *Review of Economics and Statistics*, 2013, 95(3), 776-797.
[20] Engle, R., and Kelly, B., "Dynamic Equicorrelation", *Journal of Business and Economic Statistics*, 2012, 30(2), 212-228.
[21] Engle, R. F., and Kroner, K. F., "Multivariate Simultaneous Generalized ARCH", *Econometric Theory*, 1995, 11(1), 122-150.
[22] Engle, R. F., Ledoit, O., & Wolf, M., "Large dynamic covariance matrices", J*ournal of Business & Economic Statistics*, 2019, 37(2), 363-375.
[23] Fan, Q., Wu, R., Yang, Y., & Zhong, W., "Time-varying minimum variance portfolio", Journal of Econometrics, 2022.
[24] Fleming, J., Kirby, C., and Ostdiek, B., "The Economic Value of Volatility Timing Using "Realized" Volatility", *Journal of Financial Economics*, 2003, 67(3), 473-509.
[25] Golosnoy, V., and Gribisch, B., "Modeling and Forecasting Realized Portfolio Weights", *Journal of Banking & Finance*, 2022, 106404.
[26] Gorgi, P., Hansen, P. R., Janus, P., and Koopman, S. J., "Realized Wishart-GARCH: A Score-Driven Multi-Asset Volatility Model", *Journal of Financial Econometrics*, 2019, 17(1), 1-32.
[27] Grønborg, N. S., Lunde, A., Olesen, K. V., and Vander Elst, H., "Realizing Correlations Across Asset Classes", *Journal of Financial Markets*, 2022, 59, 100729.
[28] Hansen, P. R., Lunde, A., and Nason, J. M., "The Model Confidence Set", *Econometrica*, 2011, 79(2), 453-497.
[29] Hansen, P. R., Huang, Z., and Shek, H. H., "Realized GARCH: A Joint Model for Returns and Realized Measures of Volatility", *Journal of Applied Econometrics*, 2012, 27(6), 877-906.
[30] Laurent, Sébastien, Jeroen VK Rombouts, and Francesco Violante., "On Loss Functions and Ranking Forecasting Performances of Multivariate Volatility Models", Journal of Econometrics, 173(1), 2013, 1-10.
[31] Longerstaey, J., and Spencer, M., "Riskmetricstm—Technical Document", Morgan Guaranty Trust Company of New York: New York, 1996, 51, 54.
[32] Lyócsa, Š., Molnár, P., & Výrost, T., "Stock Market Volatility Forecasting: Do We Need High-Frequency Data?", *International Journal of Forecasting*, 2021, 37(3), 1092-1110.
[33] Maheu, J. M., and McCurdy, T. H., "Do High-Frequency Measures of Volatility Improve Forecasts of Return Distributions?", *Journal of Econometrics*, 2011, 160(1), 69-76.
[34] Martens, M., "Forecasting Daily Exchange Rate Volatility Using Intraday Returns", *Journal of International Money and Finance*, 2001, 20(1), 1-23.
[35] Noureldin, D., Shephard, N., and Sheppard, K., "Multivariate High - Frequency - Based Volatility (HEAVY) Models", *Journal of Applied Econometrics*, 2012, 27(6), 907-933.
[36] Patton, A. J., "Volatility Forecast Comparison Using Imperfect Volatility Proxies", *Journal of Econometrics*, 2011, 160(1), 246-256.
[37] Patton, Andrew J., and Kevin Sheppard., "Evaluating Volatility and Correlation Forecasts", Handbook of financial time series. Springer, Berlin, Heidelberg, 2009. 801-838.
[38] Pakel, C., Shephard, N., Sheppard, K., and Engle, R. F., "Fitting Vast Dimensional Time-Varying Covariance Models", *Journal of Business & Economic Statistics*, 2021, 39(3), 652-668.
[39] Shephard N, and Sheppard K., "Realising the Future: Forecasting with High-Frequency-Based Volatility (HEAVY) Models", *Journal of Applied Econometrics*, 2010, 25(2): 197-231.